Universal exploration dynamics of random walks

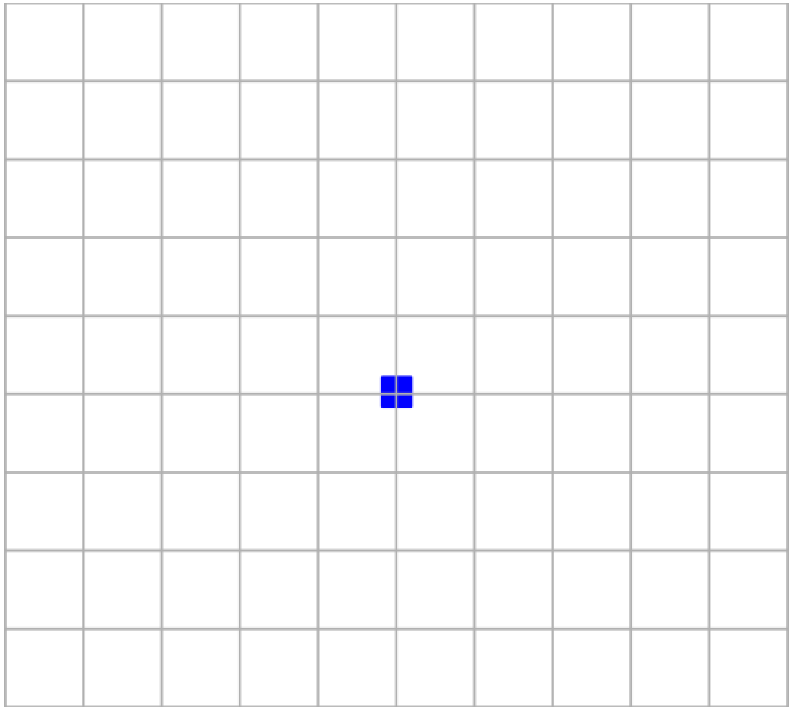
Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



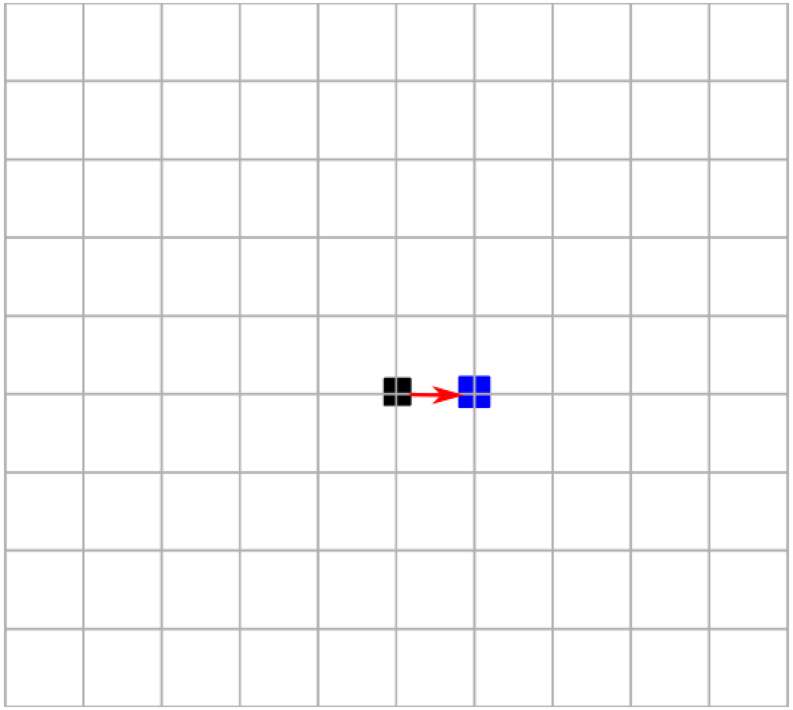
■ : visited sites.

■ : current position.

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



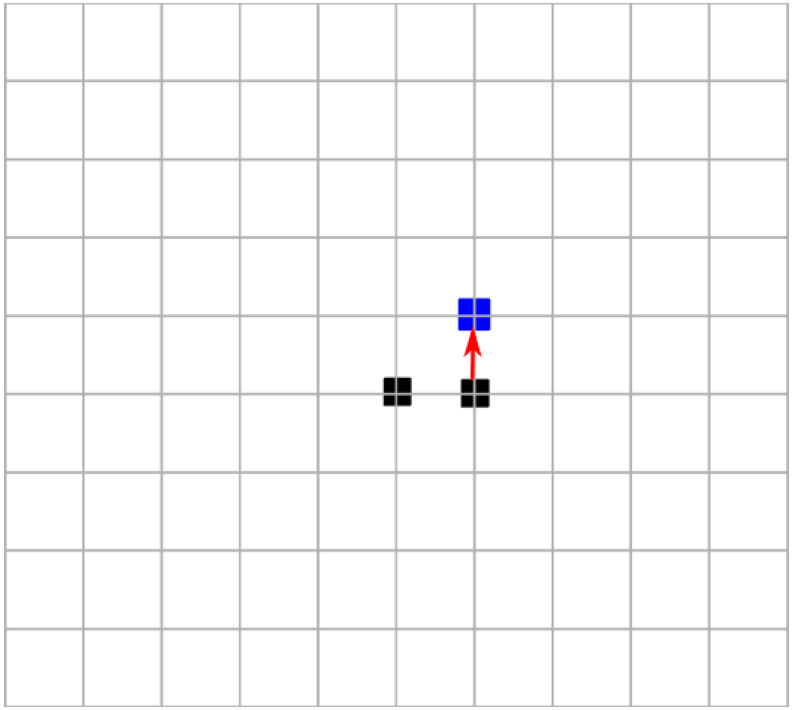
■ : visited sites.

■ : current position.

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



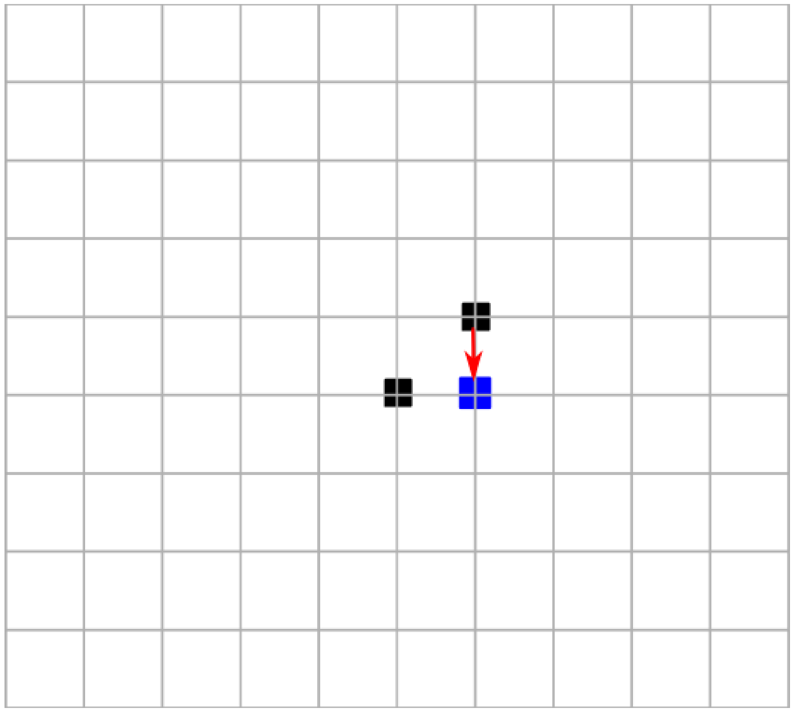
: visited sites.

: current position.

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



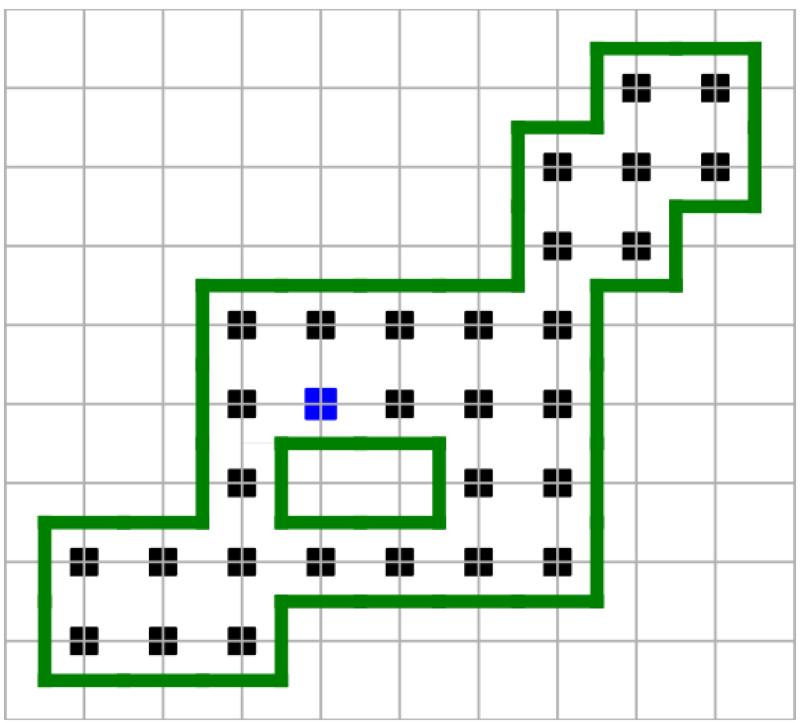
■ : visited sites.

■ : current position.

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



■ : visited sites.

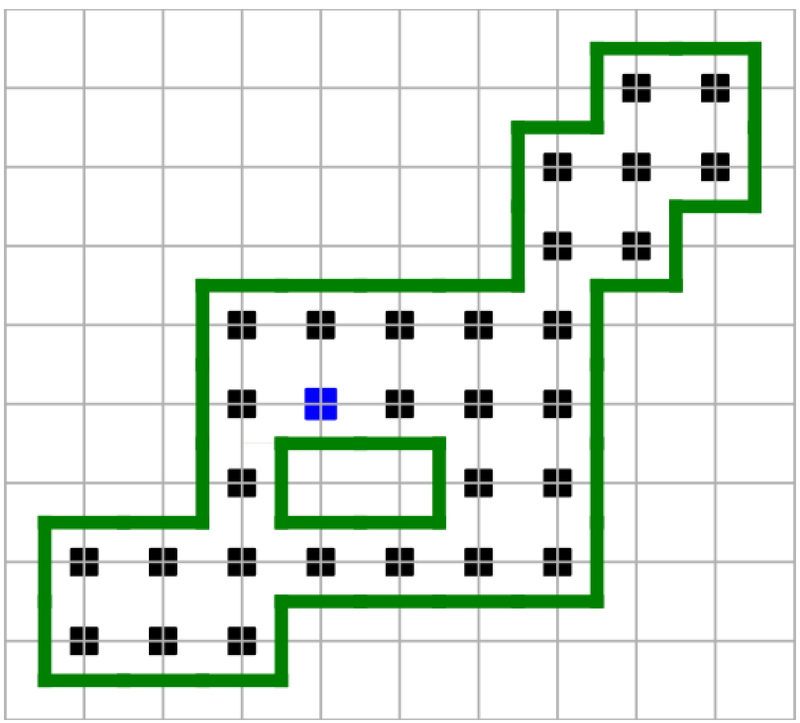
| : boundary between visited/univisited sites.

■ : current position, *n*th visited site.

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



■ : visited sites.

| : boundary between visited/univisited sites.

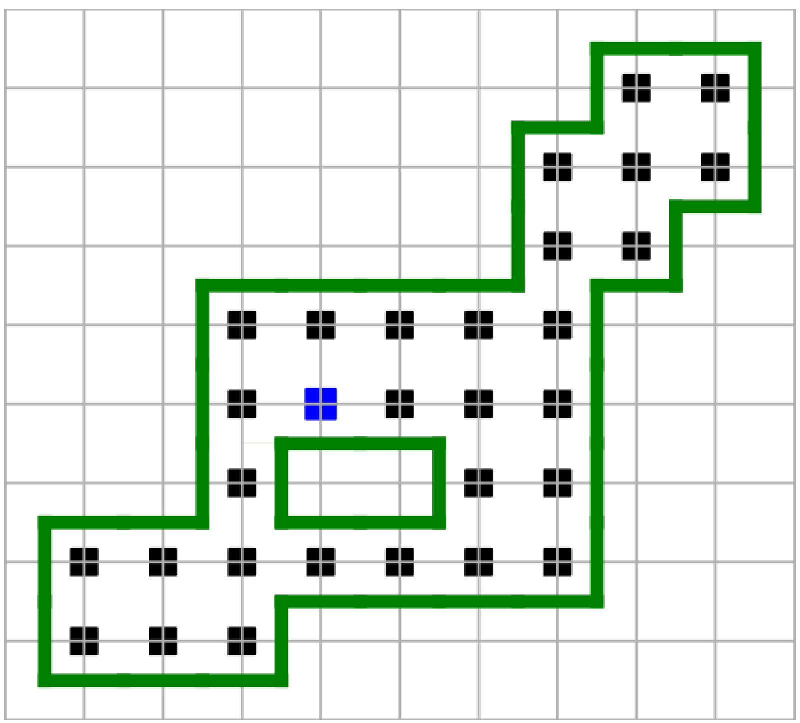
■ : current position, *n*th visited site.

Exploration: usually quantified by the number of distinct sites visited at time t, N(t)

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



■ : visited sites.

| : boundary between visited/univisited sites.

■ : current position, *n*th visited site.

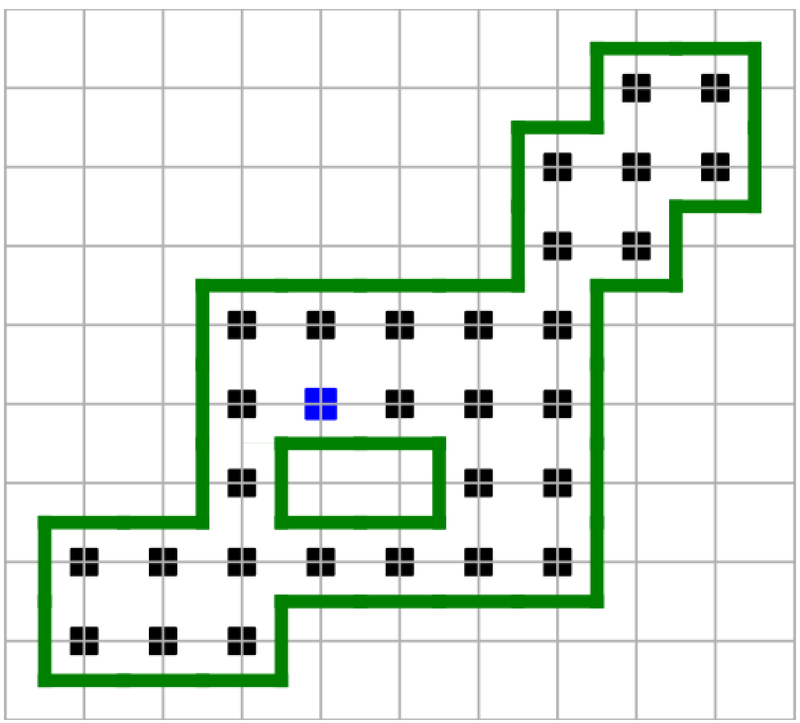
Exploration: usually quantified by the number of distinct sites visited at time t, N(t)

→ Average, variance and distribution

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



■ : visited sites.

| : boundary between visited/univisited sites.

■ : current position, *n*th visited site.

Exploration: usually quantified by the number of distinct sites visited at time t, N(t)

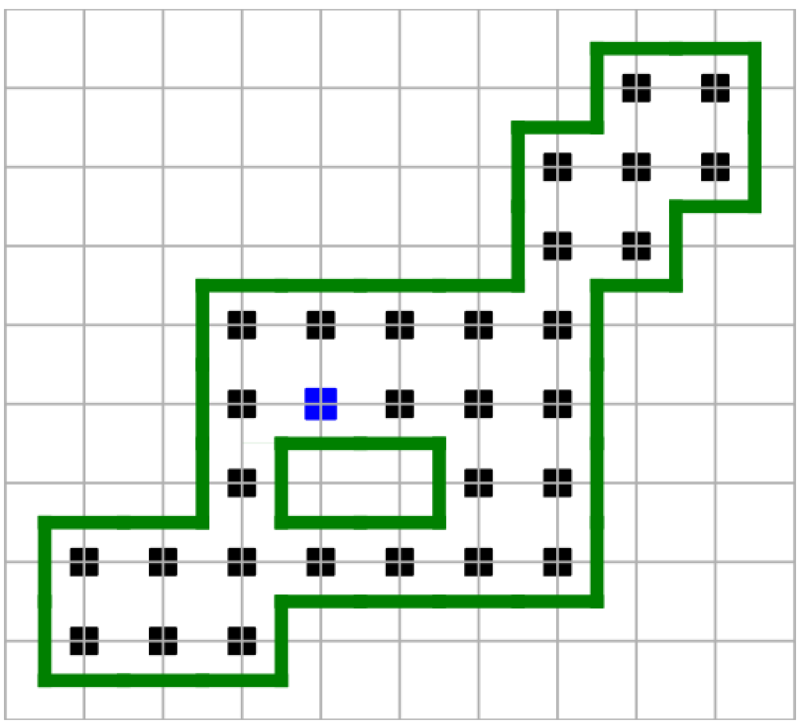
→ Average, variance and distribution

→ Foraging: food collected

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



■ : visited sites.

| : boundary between visited/univisited sites.

■ : current position, *n*th visited site.

Exploration: usually quantified by the number of distinct sites visited at time t, N(t)

→ Average, variance and distribution

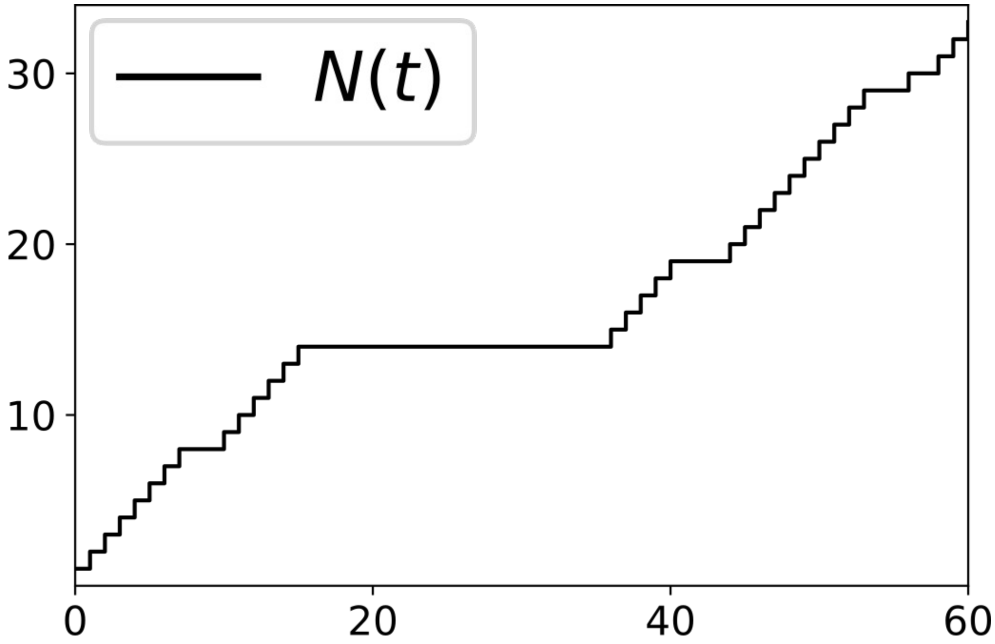
→ Foraging: food collected

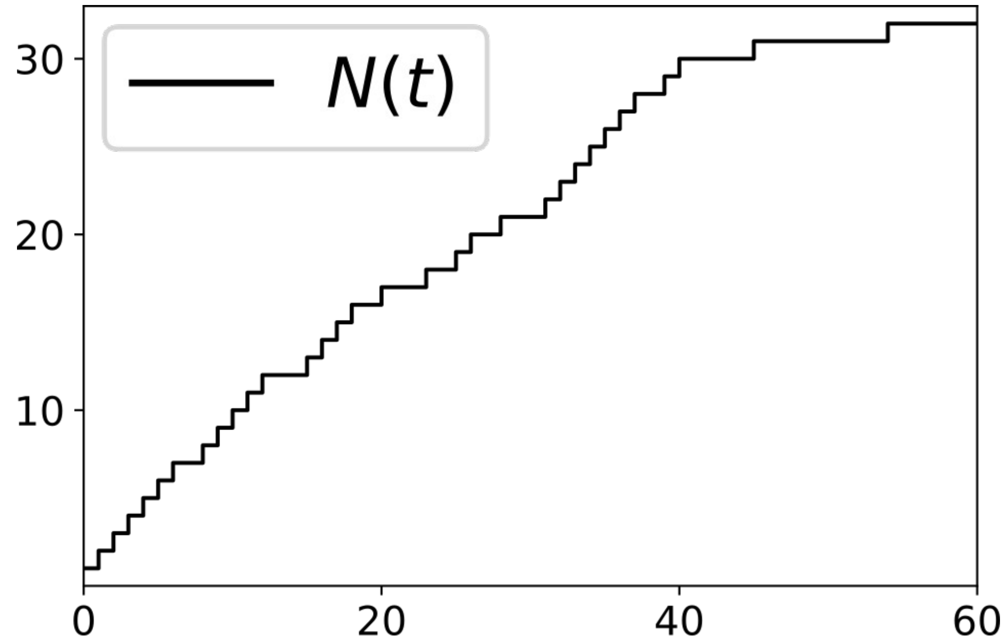
→ Trapping problem: trap concentration p, Survival probability =<$(1-p)^{N(t)}$ >

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)

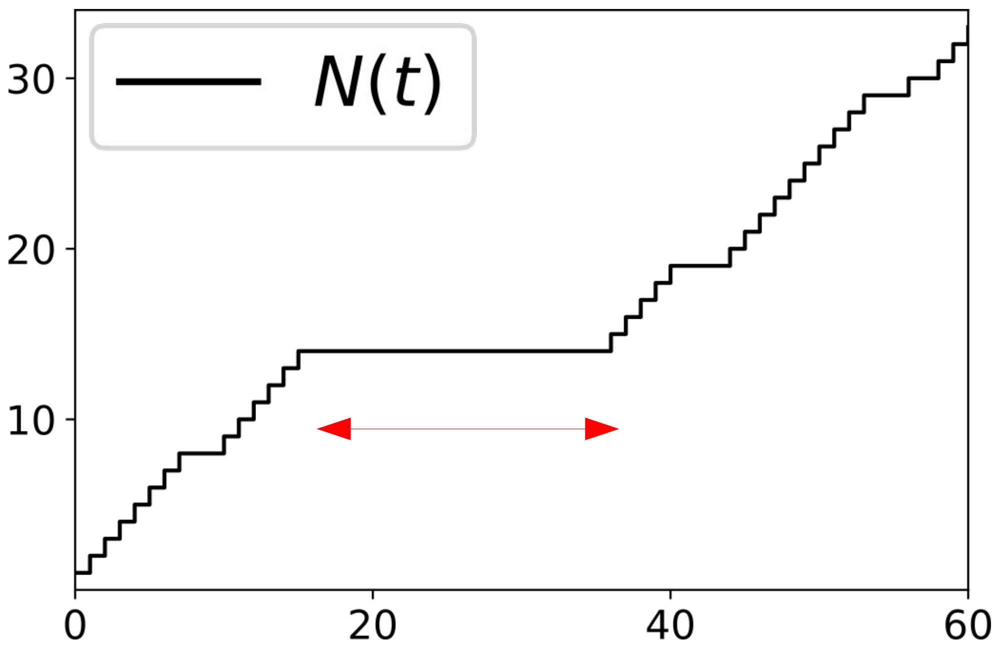




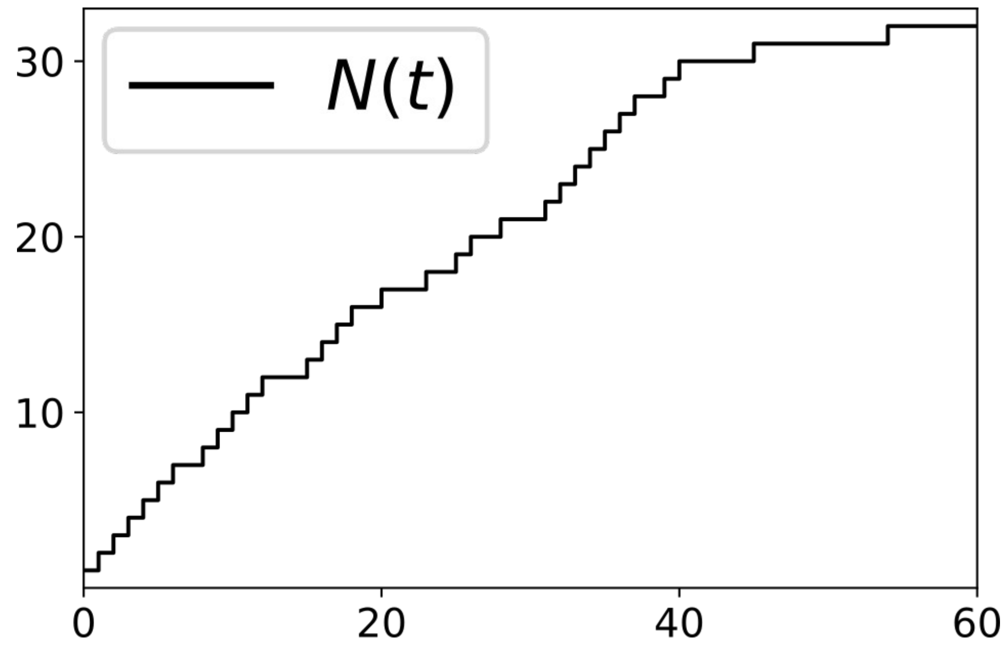
Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



Irregular food intake [starves]

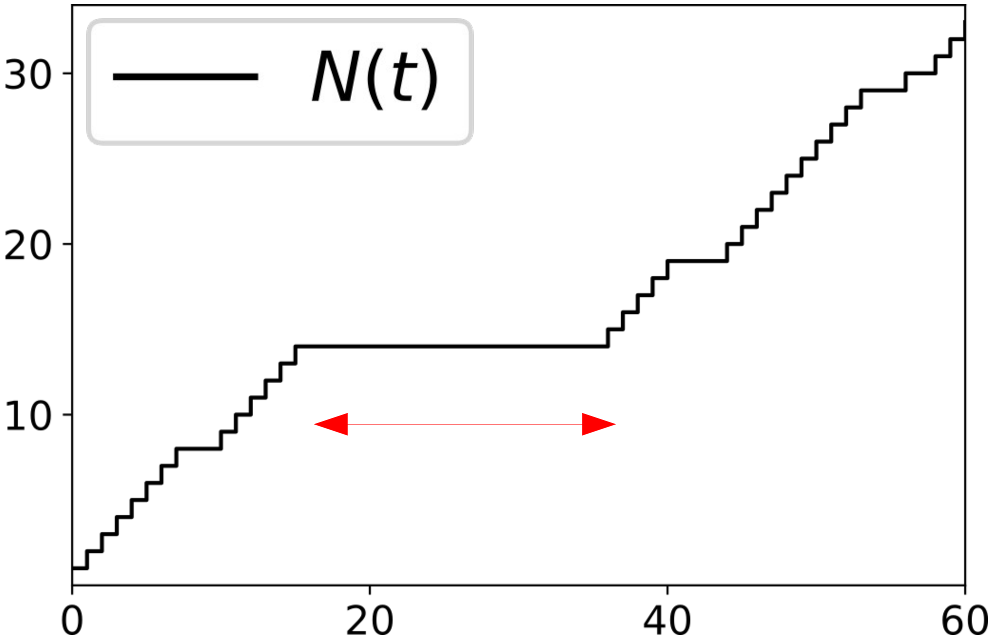


Regular food intake [survives]

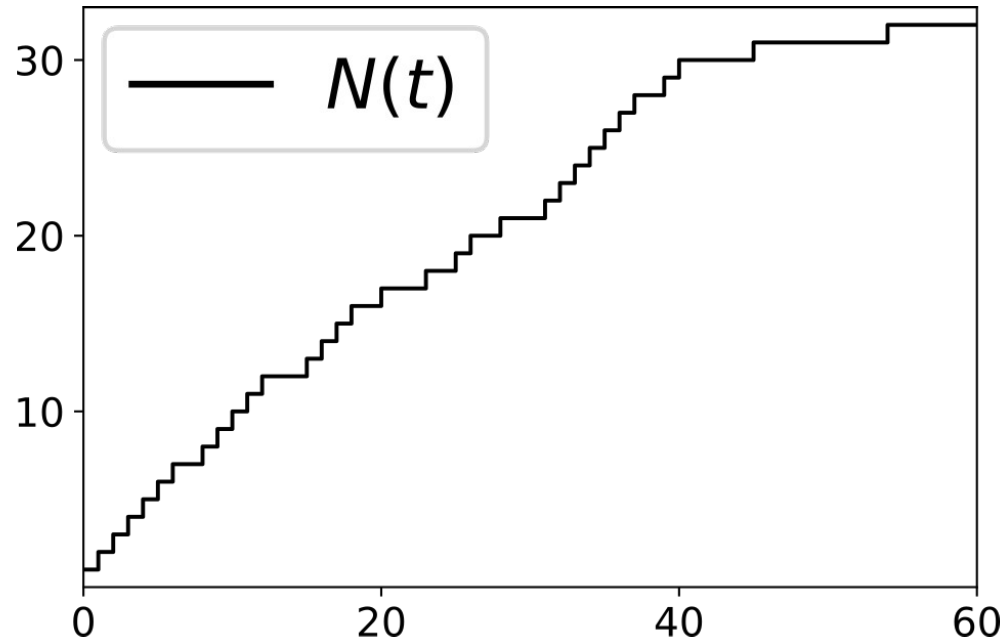
Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



Irregular food intake [starves]



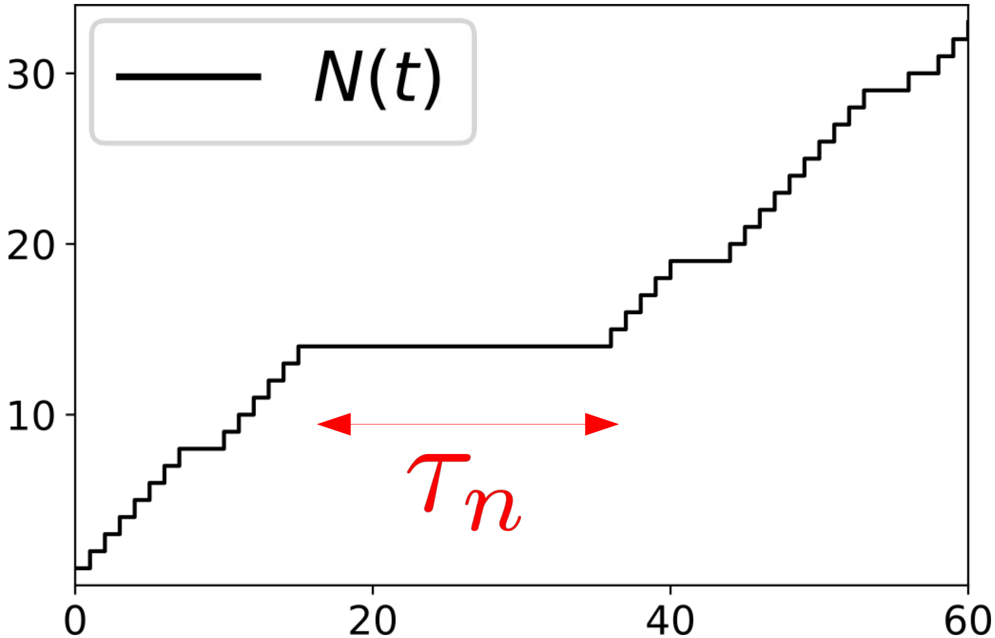
Regular food intake [survives]

Time elapsed between finding of new resources?

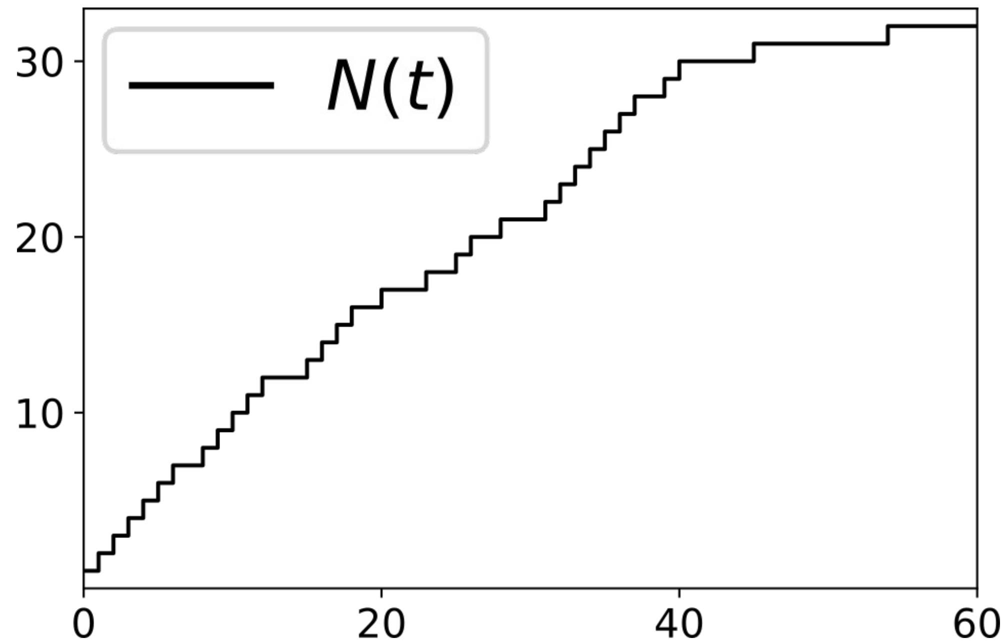
Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)



Irregular food intake [starves]



Regular food intake [survives]

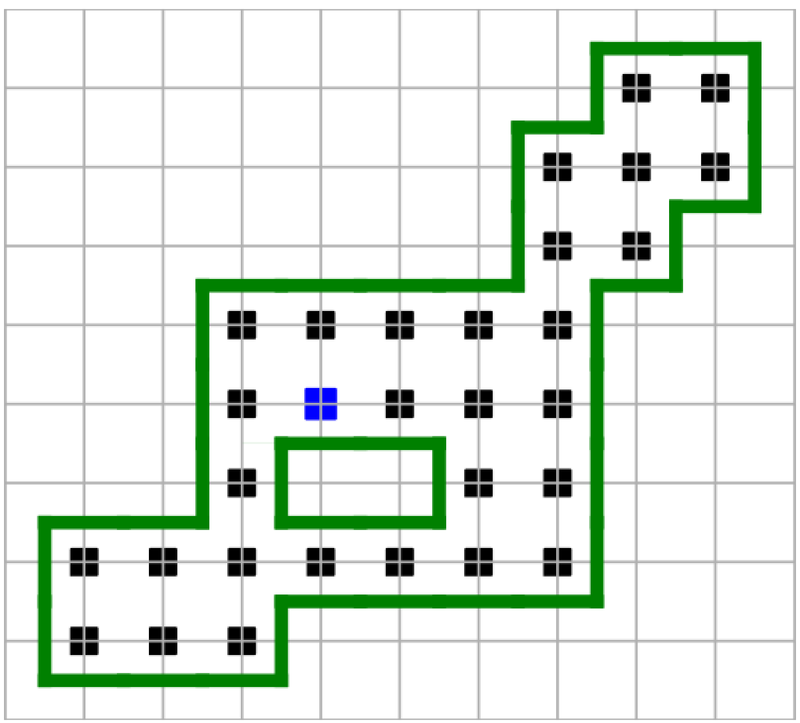
Time elapsed between finding of new resources?

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)

A random walker (RW) has visited n distinct sites:
How long does it take to visit a new site?



■ : visited sites.

| : boundary between visited/univisited sites.

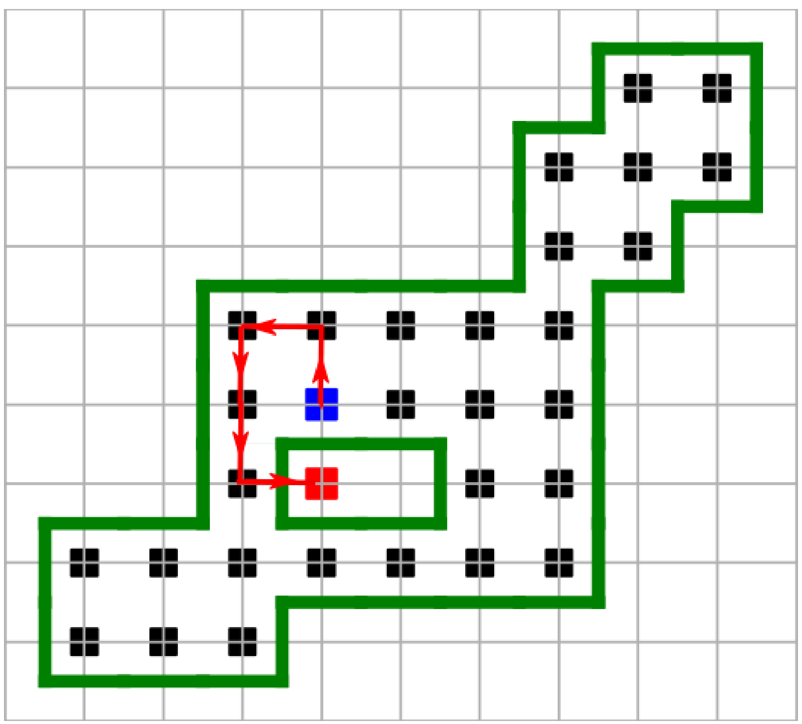
■ : nth visited site.

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)

A random walker (RW) has visited n distinct sites:
How long does it take to visit a new site?



■ : visited sites.

| : boundary between visited/univisited sites.

■ : nth visited site. ■ : $(n+1)$st visited site.

$\mathcal{T}_n$ = time elapsed between the visit of the nth and the $(n+1)$st new sites [distribution F_n].

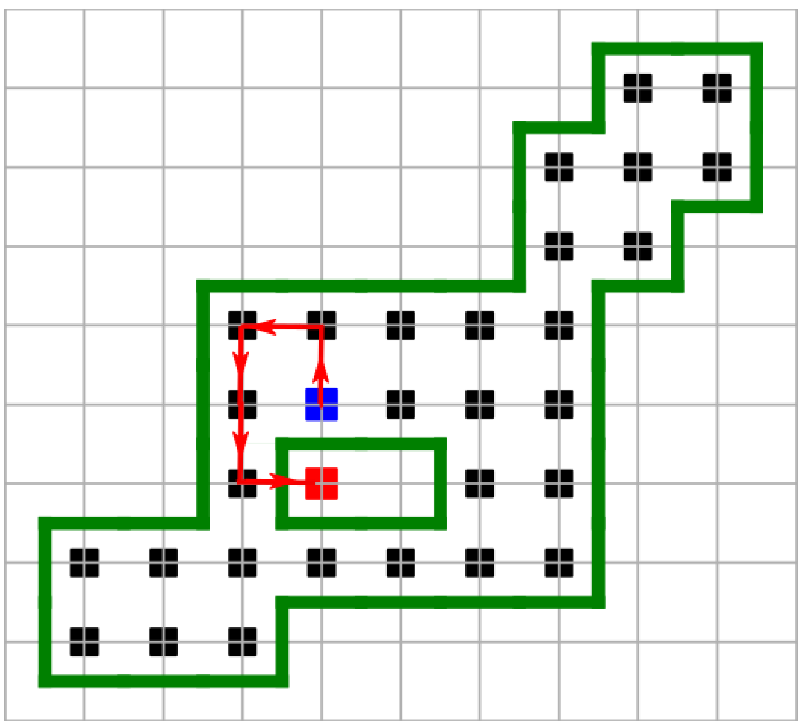
(here, n =30 and $\mathcal{T}_n$=5)

Universal exploration dynamics of random walks

Léo Régnier
M. Dolgushev
S. Redner
O. Bénichou

Nat. Commun. **14**, 618, (2023)

A random walker (RW) has visited n distinct sites:
How long does it take to visit a new site?



■ : visited sites.

| : boundary between visited/univisited sites.

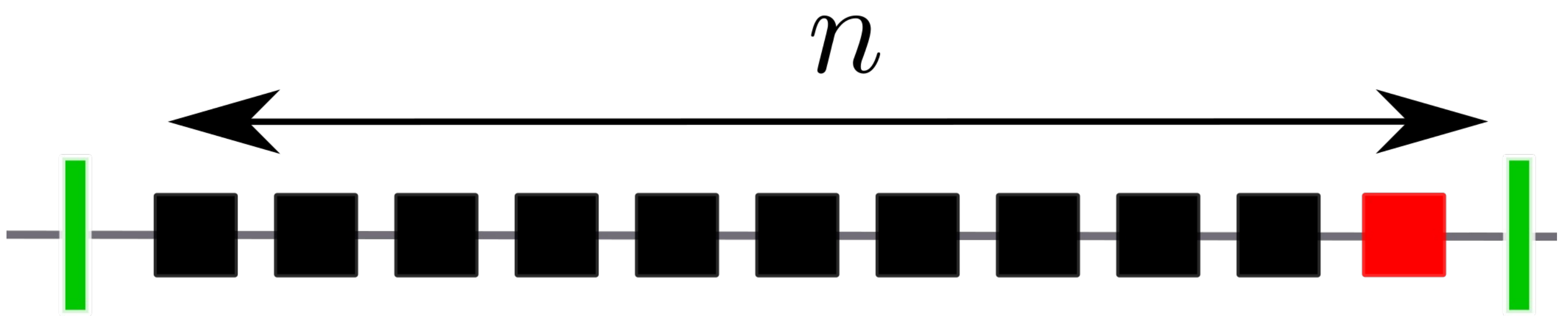
■ : nth visited site. ■ : $(n+1)$st visited site.

τ_n = time elapsed between the visit of the nth and the $(n+1)$st new sites [distribution F_n].

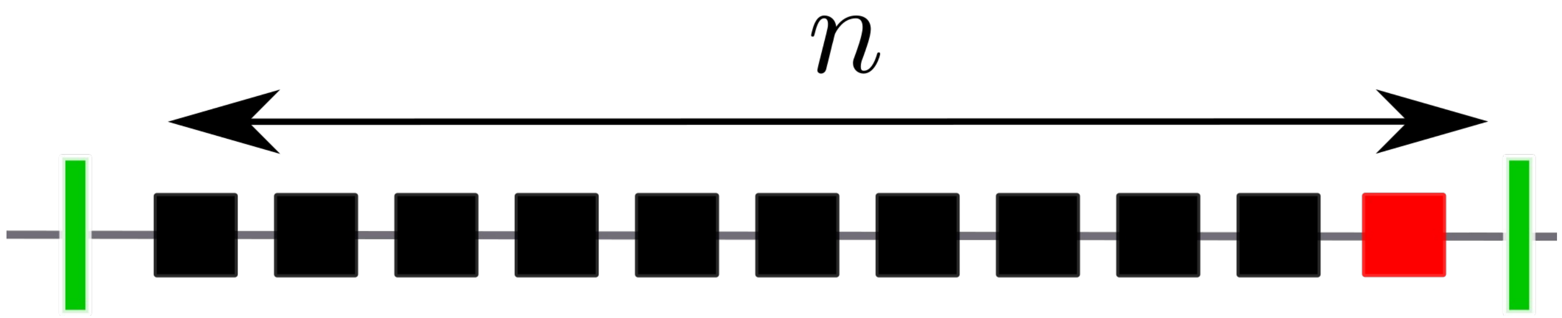
(here, n =30 and τ_n=5)

Objective: F_n characterization ? n dependence?

The 1d case



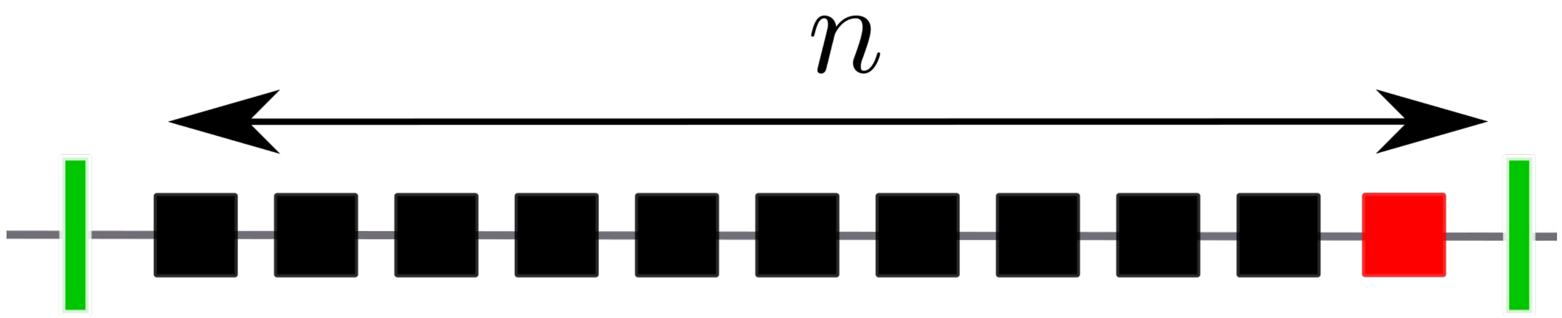
The 1d case



$F_n(\tau)$ is the first exit time probability starting one lattice step from the boundary

$$F_n(\tau) \sim \frac{2\pi^2}{n^3} \sum_{k=0}^{\infty} (2k+1)^2 \exp\left[-\pi^2 (2k+1)^2 \tau / 2n^2\right]$$

The 1d case



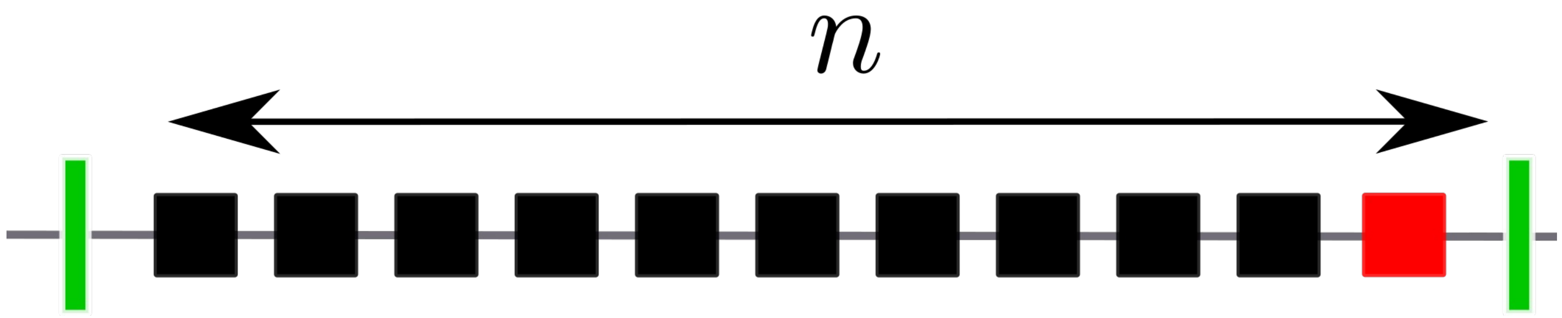
$F_n(\tau)$ is the first exit time probability starting one lattice step from the boundary

$$F_n(\tau) \sim \frac{2\pi^2}{n^3} \sum_{k=0}^{\infty} (2k+1)^2 \exp\left[-\pi^2(2k+1)^2\tau/2n^2\right]$$

Properties:

(i) Depends on n, aging

The 1d case



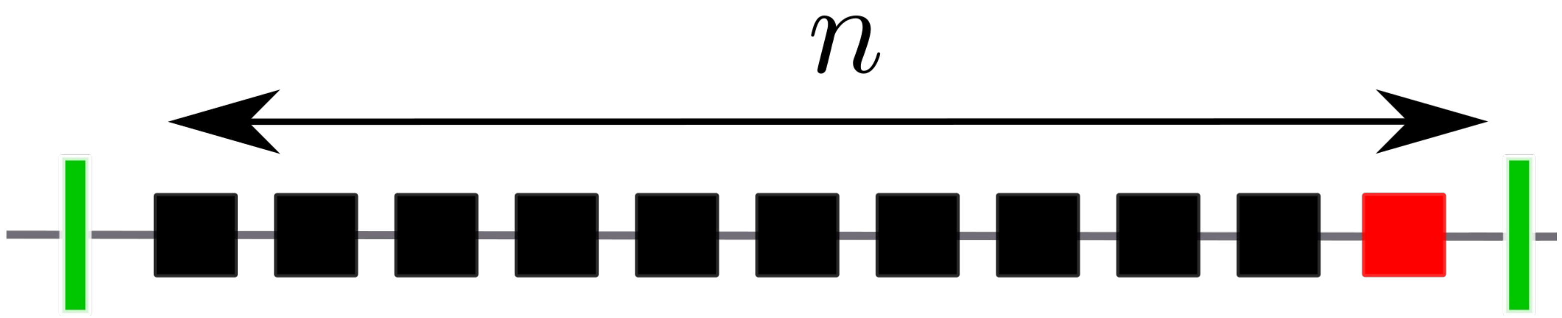
$F_n(\tau)$ is the first exit time probability starting one lattice step from the boundary

$$F_n(\tau) \sim \frac{2\pi^2}{n^3} \sum_{k=0}^{\infty} (2k+1)^2 \exp\left[-\pi^2 (2k+1)^2 \tau / 2n^2\right]$$

Properties:

(i) Depends on n, aging
(ii) Algebraic at small times, $\tau^{-3/2}$

The 1d case



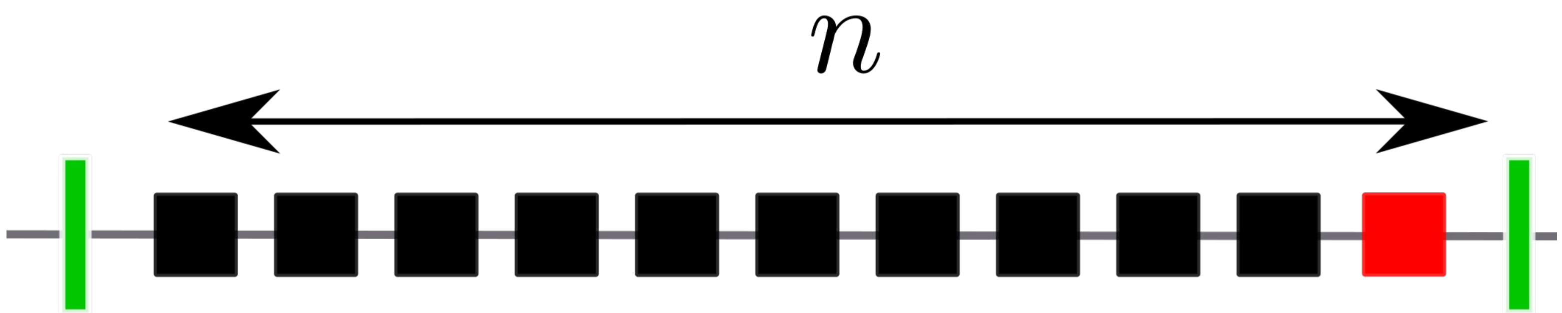
$F_n(\tau)$ is the first exit time probability starting one lattice step from the boundary

$$F_n(\tau) \sim \frac{2\pi^2}{n^3} \sum_{k=0}^{\infty} (2k+1)^2 \exp\left[-\pi^2 (2k+1)^2 \tau / 2n^2\right]$$

Properties:

(i) Depends on n, aging
(ii) Algebraic at small times, $\tau^{-3/2}$
(iii) Exponential decay at large times

The 1d case



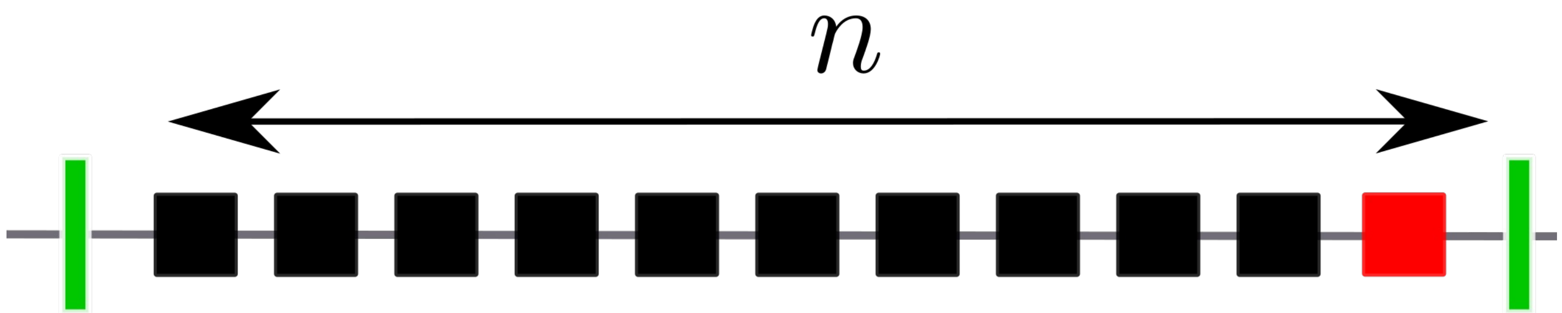
$F_n(\tau)$ is the first exit time probability starting one lattice step from the boundary

$$F_n(\tau) \sim \frac{2\pi^2}{n^3} \sum_{k=0}^{\infty} (2k+1)^2 \exp\left[-\pi^2 (2k+1)^2 \tau / 2n^2\right]$$

Properties:

(i) Depends on n, aging
(ii) Algebraic at small times, $\tau^{-3/2}$
(iii) Exponential decay at large times
(iv) Scaling form, $F_n(\tau) = n^{-3} \psi(\tau / n^2)$

The 1d case



$F_n(\tau)$ is the first exit time probability starting one lattice step from the boundary

$$F_n(\tau) \sim \frac{2\pi^2}{n^3} \sum_{k=0}^{\infty} (2k+1)^2 \exp\left[-\pi^2(2k+1)^2\tau/2n^2\right]$$

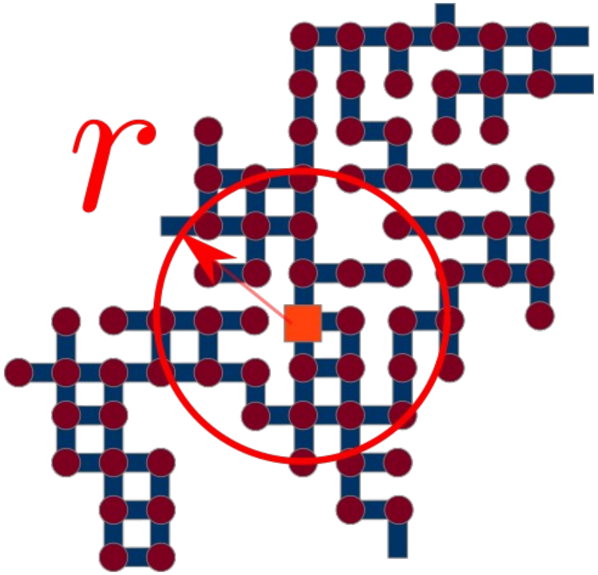
Properties:

(i) Depends on n, aging
(ii) Algebraic at small times, $\tau^{-3/2}$
(iii) Exponential decay at large times
(iv) Scaling form, $F_n(\tau) = n^{-3}\psi(\tau/n^2)$

Question:

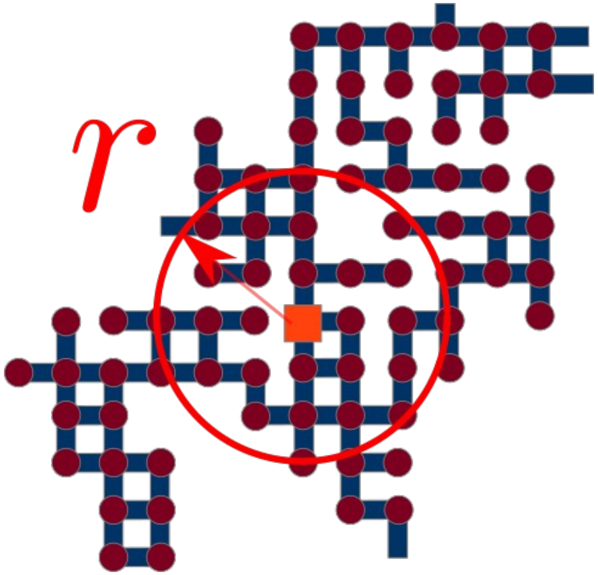
What about more general RWs in more complex geometries?

We consider symmetric, Markovian RWs in discrete time + scale-invariance



Defining general RWs

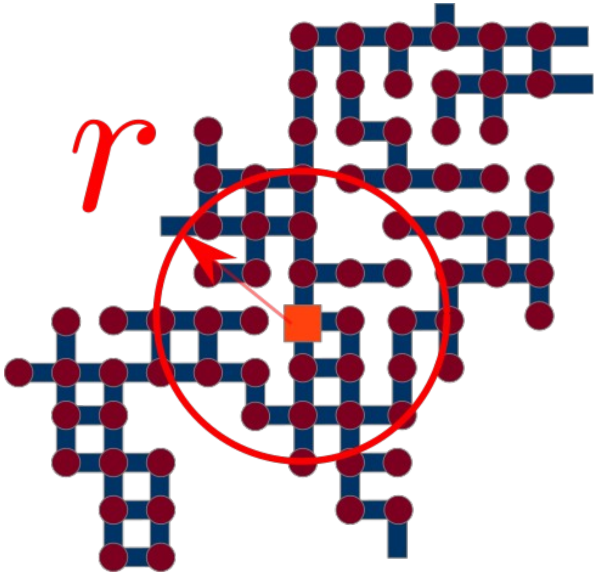
We consider symmetric, Markovian RWs in discrete time + scale-invariance



Number of sites within a circle of radius r (d_f = fractal dimension, d for lattice) $\propto r^{d_{\rm f}}$

Defining general RWs

We consider symmetric, Markovian RWs in discrete time + scale-invariance

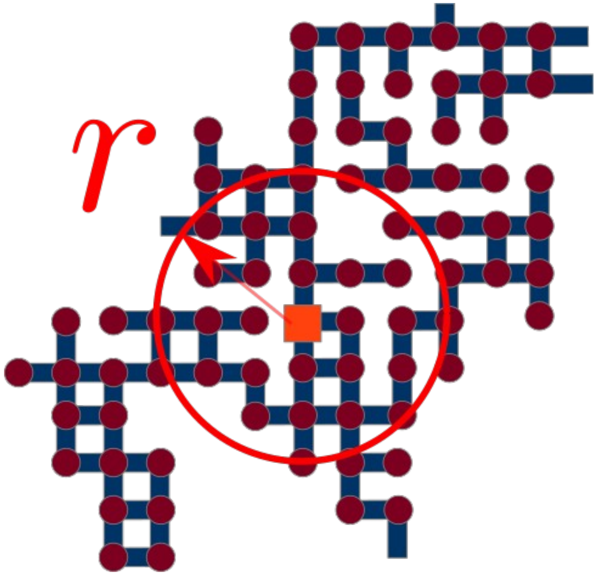


Number of sites within a circle of radius r
(d_f = fractal dimension, d for lattice) $\propto r^{d_\mathrm{f}}$

Exit time of a circle of radius r
(d_w=walk dimension, 2 for diffusion) $\propto r^{d_\mathrm{w}}$

Defining general RWs

We consider symmetric, Markovian RWs in discrete time + scale-invariance



Number of sites within a circle of radius r (d_f = fractal dimension, d for lattice) $\propto r^{d_{\mathrm{f}}}$

Exit time of a circle of radius r (d_w=walk dimension, 2 for diffusion) $\propto r^{d_{\mathrm{w}}}$

$$\mu \equiv \frac{d_{\mathrm{f}}}{d_{\mathrm{w}}}$$

Recurrent: <1

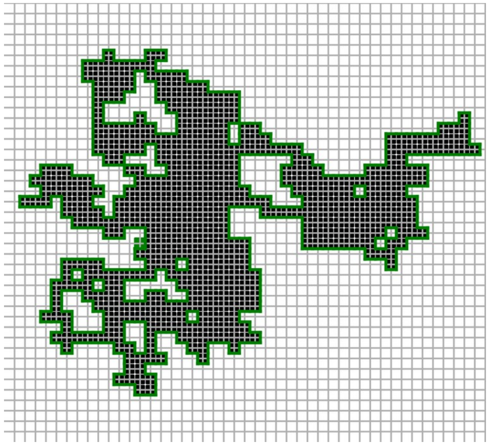
Marginal: =1

Transient: >1

Mapping to a trapping problem

METHOD:

Mapping with a trapping problem where traps=non-visited sites.

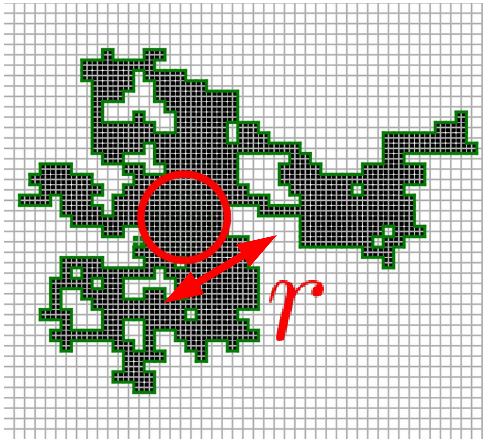


Mapping to a trapping problem

METHOD:

Mapping with a trapping problem where traps=non-visited sites.

$Q_n(r)$ =Distribution of the radius of the largest spherical region free of traps (for *n* visited sites)

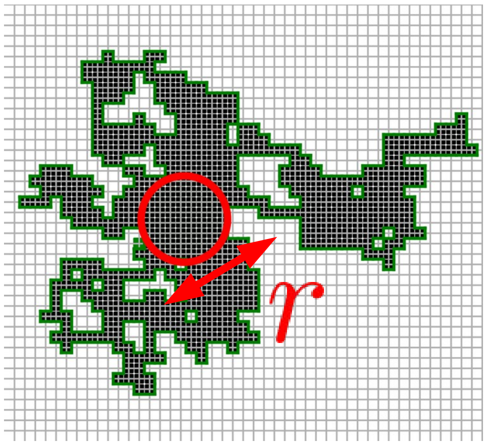


Mapping to a trapping problem

METHOD:

Mapping with a trapping problem where traps=non-visited sites.

$Q_n(r)$ =Distribution of the radius of the largest spherical region free of traps (for *n* visited sites)



We obtain via scaling arguments,

$$Q_n(r) \approx \rho_n^{-1} \exp\left[-a(r/\rho_n)^{d_{\mathrm{f}}}\right]$$

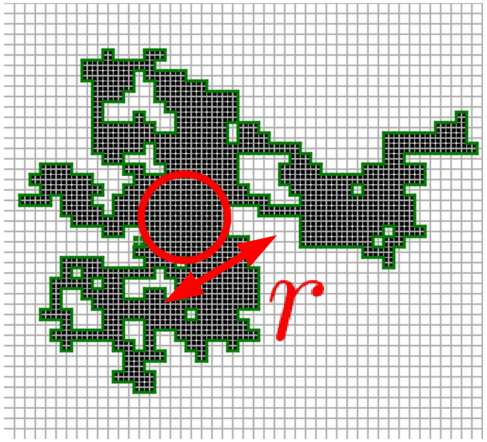
ρ_n provides the typical scale of the largest fully visited region,

Mapping to a trapping problem

METHOD:

Mapping with a trapping problem where traps=non-visited sites.

$Q_n(r)$ =Distribution of the radius of the largest spherical region free of traps (for *n* visited sites)



We obtain via scaling arguments,

$$Q_n(r) \approx \rho_n^{-1} \exp\left[-a(r/\rho_n)^{d_\mathrm{f}}\right]$$

ρ_n provides the typical scale of the largest fully visited region,

Different from classical trapping problem:

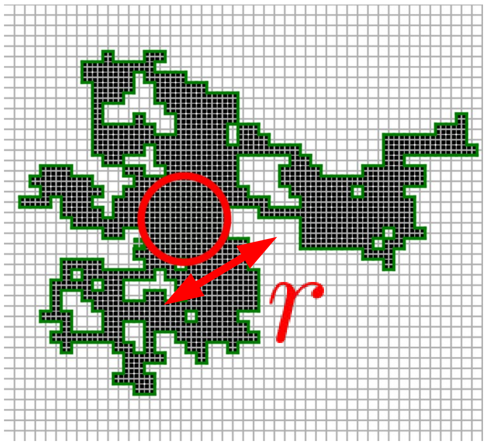
- Aging
- Correlations

Mapping to a trapping problem

METHOD:

Mapping with a trapping problem where traps=non-visited sites.

$Q_n(r)$ =Distribution of the radius of the largest spherical region free of traps (for n visited sites)



We obtain via scaling arguments,

$$Q_n(r) \approx \rho_n^{-1} \exp\left[-a(r/\rho_n)^{d_\mathrm{f}}\right]$$

ρ_n provides the typical scale of the largest fully visited region,

$$\rho_n \propto \begin{cases} n^{1/d_\mathrm{f}}, \text{ if } \mu < 1 \\ \sqrt{n^{1/d_\mathrm{f}}}, \text{ if } \mu = 1 \\ 1, \text{ if } \mu > 1 \end{cases}$$

Different from classical trapping problem:

- Aging
- Correlations

Results:

For general scale-invariant Markovian process:

$$\mu \equiv \frac{d_{\mathrm{f}}}{d_{\mathrm{w}}}$$

Results: recurrent RWs

For general scale-invariant Markovian process:

$$\mu \equiv \frac{d_{\mathrm{f}}}{d_{\mathrm{w}}}$$

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$ [redacted]	[redacted]	$\exp\left[-\operatorname{const} \tau / n^{1/\mu} \right]$

Algebraic

Exponential

Scale invariance

Results: transient RWs

For general scale-invariant Markovian process:

$$\mu \equiv \frac{d_{\mathrm{f}}}{d_{\mathrm{w}}}$$

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$	[redacted]	[redacted] $\exp\left[-\,\mathrm{const}\,(\tau/t_n)^{\mu/(1+\mu)} \right]$	$\exp\left[-\,\mathrm{const}\ \tau/n^{1/\mu} \right]$

Stretched exponential

Exponential

For general scale-invariant Markovian process:

$$\mu \equiv \frac{d_\mathrm{f}}{d_\mathrm{w}}$$

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$	$\tau^{-(1+\mu)}$ [redacted]	[redacted] $\exp\left[-\mathrm{const}\,(\tau/t_n)^{\mu/(1+\mu)}\right]$	$\exp\left[-\mathrm{const}\,\tau/n^{1/\mu}\right]$

Algebraic

Stretched exponential

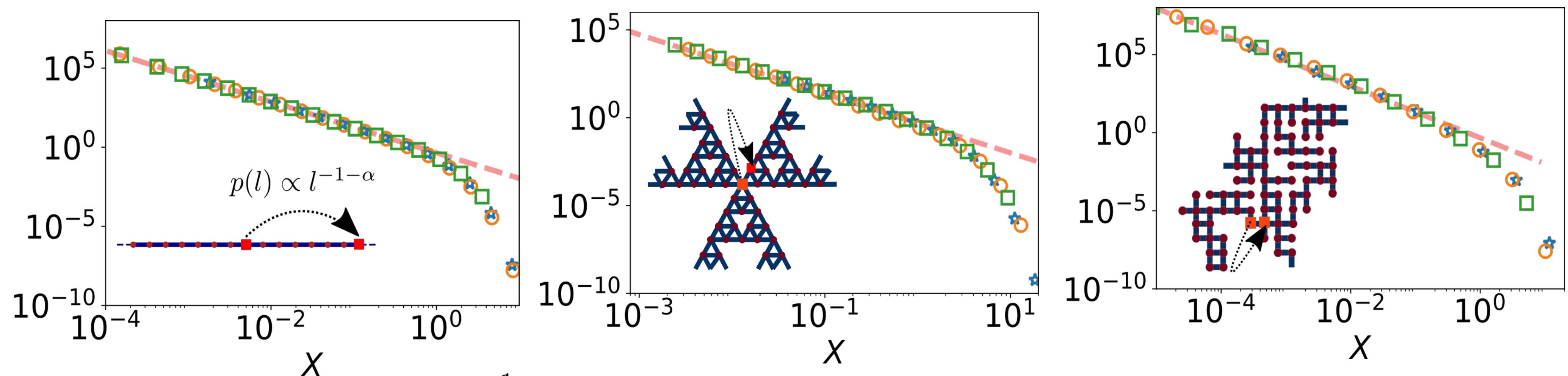
Exponential

For general scale-invariant Markovian process:

$$\mu \equiv \frac{d_{\mathrm{f}}}{d_{\mathrm{w}}}$$

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$		
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$		$\exp\left[-\,\mathrm{const}\, (\tau/t_n)^{\mu/(1+\mu)} \right]$	$\exp\left[-\,\mathrm{const}\; \tau/n^{1/\mu} \right]$
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$			

Distribution F_n of the inter visit time: Recurrent



1d Lévy flight $p(\ell) \propto \ell^{-1-\alpha}$	Sierpinski gasket	Percolation cluster
$\mu = 1/\alpha < 1$	$\mu = \ln 3/\ln 5$	$\mu \approx 0.659$
[superdiffusive, long jumps]	[subdiffusive, fractal]	[subdiffusive, disordered]

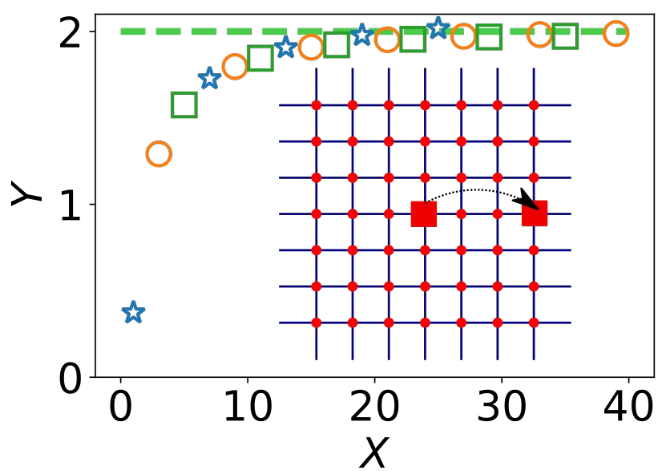
$$X = \tau/t_n,\ Y = F_n(\tau)t_n^{\mu+1}$$

Algebraic

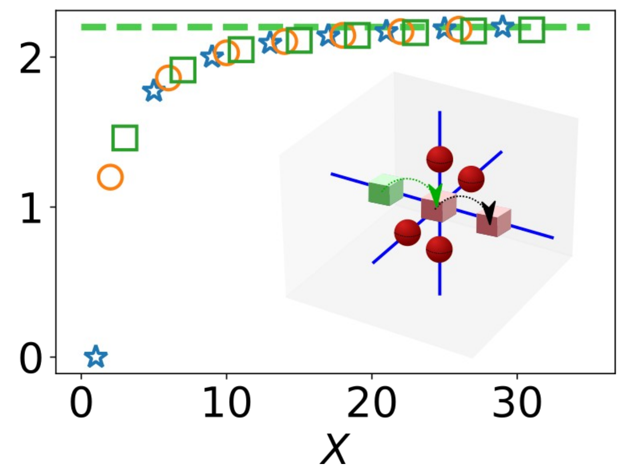
Exponential

Scale invariance

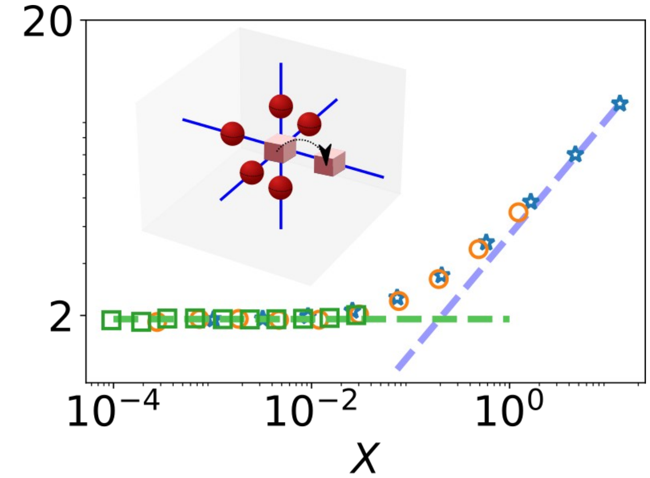
Distribution F_n of the inter visit time: Transient



2d Lévy flight $p(\ell) \propto \ell^{-1-\alpha}$



3d persistent RW



3d simple RW

$$\mu = 2/\alpha > 1 \qquad \mu = 3/2 \qquad \mu = 3/2$$

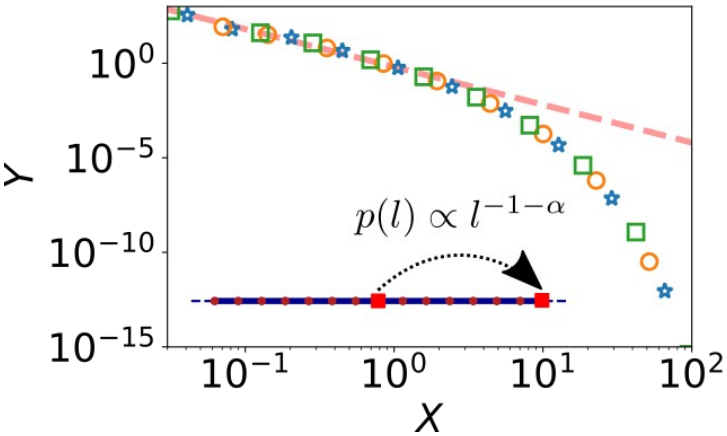
$$X = \tau \qquad X = \tau \qquad X = \tau/T_n$$

$$Y = -\ln[F_n(\tau)]/(\tau/t_n)^{\mu/(1+\mu)}$$

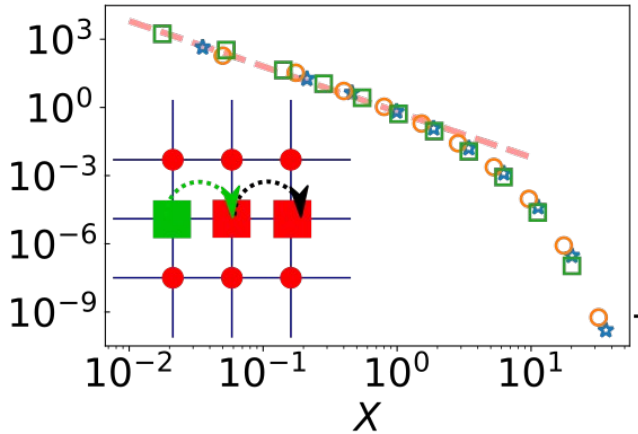
Stretched exponential

Exponential

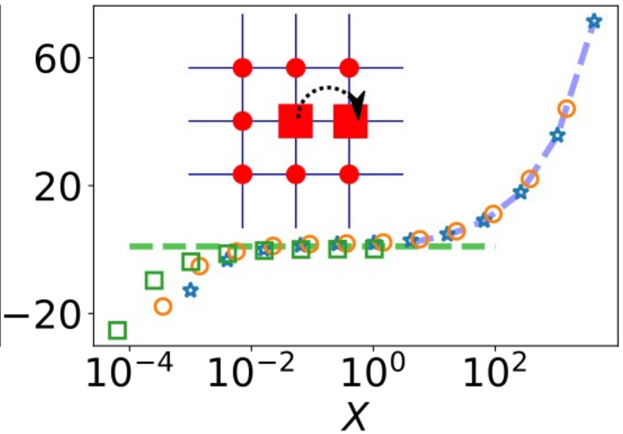
Distribution F_n of the inter visit time: Marginal



1d Lévy flight



2d persistent RW



2d simple RW

$$X = \tau/t_n,\ Y = F_n(\tau)t_n^{\mu+1}$$

$$X = \tau/T_n$$

$$Y = -\ln[F_n(\tau)]/(\tau/t_n)^{\mu/(1+\mu)}$$

Algebraic

Stretched exponential

Exponential

For general scale-invariant Markovian process:

<table>
<tr><th></th><th>t_n</th><th>T_n</th><th>$1 \ll \tau \ll t_n$</th><th>$t_n \ll \tau \ll T_n$</th><th>$T_n \ll \tau$</th></tr>
<tr><td>$\mu < 1$ [recurrent]</td><td>$n^{1/\mu}$</td><td>$n^{1/\mu}$</td><td rowspan="2">$\tau^{-(1+\mu)}$</td><td></td><td rowspan="3">$\exp\left[-\text{const}\ \tau/n^{1/\mu} \right]$</td></tr>
<tr><td>$\mu = 1$ [marginal]</td><td>$\sqrt{n}$</td><td>$n^{3/2}$</td><td rowspan="2">$\exp\left[-\text{const}\ (\tau/t_n)^{\mu/(1+\mu)} \right]$</td></tr>
<tr><td>$\mu > 1$ [transient]</td><td>1</td><td>$n^{(\mu+1)/\mu}$</td><td></td></tr>
</table>

Conclusion: Universality

For general scale-invariant Markovian process:

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$		$\exp\left[-\text{const}\ \tau/n^{1/\mu}\right]$
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$		$\exp\left[-\text{const}\ (\tau/t_n)^{\mu/(1+\mu)}\right]$	
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$			

Complete characterisation of the time elapsed between visits for random walks:

Conclusion: Universality

For general scale-invariant Markovian process:

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$	[redacted]	
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$		$\exp\left[-\text{const}\,(\tau/t_n)^{\mu/(1+\mu)}\right]$	$\exp\left[-\text{const}\,\tau/n^{1/\mu}\right]$
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$	[redacted]		

Complete characterisation of the time elapsed between visits for random walks:

- In any dimension (3d, ...)

Conclusion: Universality

For general scale-invariant Markovian process:

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$	[redacted]	
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$			$\exp\left[-\text{const}\ \tau/n^{1/\mu}\right]$
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$	[redacted]	$\exp\left[-\text{const}\ (\tau/t_n)^{\mu/(1+\mu)}\right]$	

Complete characterisation of the time elapsed between visits for random walks:

- In any dimension (3d, ...)
- on disordered media (Percolation clusters, ...)

Conclusion: Universality

For general scale-invariant Markovian process:

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$	[redacted]	$\exp\left[-\text{const}\ \tau/n^{1/\mu}\right]$
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$	$\tau^{-(1+\mu)}$	$\exp\left[-\text{const}\ (\tau/t_n)^{\mu/(1+\mu)}\right]$	$\exp\left[-\text{const}\ \tau/n^{1/\mu}\right]$
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$	[redacted]	$\exp\left[-\text{const}\ (\tau/t_n)^{\mu/(1+\mu)}\right]$	$\exp\left[-\text{const}\ \tau/n^{1/\mu}\right]$

Complete characterisation of the time elapsed between visits for random walks:

- In any dimension (3d, ...)
- on disordered media (Percolation clusters, ...)
- on fractals (Sierpinski lattice, ...)

Conclusion: Universality

For general scale-invariant Markovian process:

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$		
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$		$\exp\left[-\text{const}\,(\tau/t_n)^{\mu/(1+\mu)}\right]$	$\exp\left[-\text{const}\,\tau/n^{1/\mu}\right]$
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$			

Complete characterisation of the time elapsed between visits for random walks:

- In any dimension (3d, ...)
- on disordered media (Percolation clusters, ...)
- on fractals (Sierpinski lattice, ...)
- with anomalous diffusion (Lévy flights, ...)

Conclusion: Universality

For general scale-invariant Markovian process:

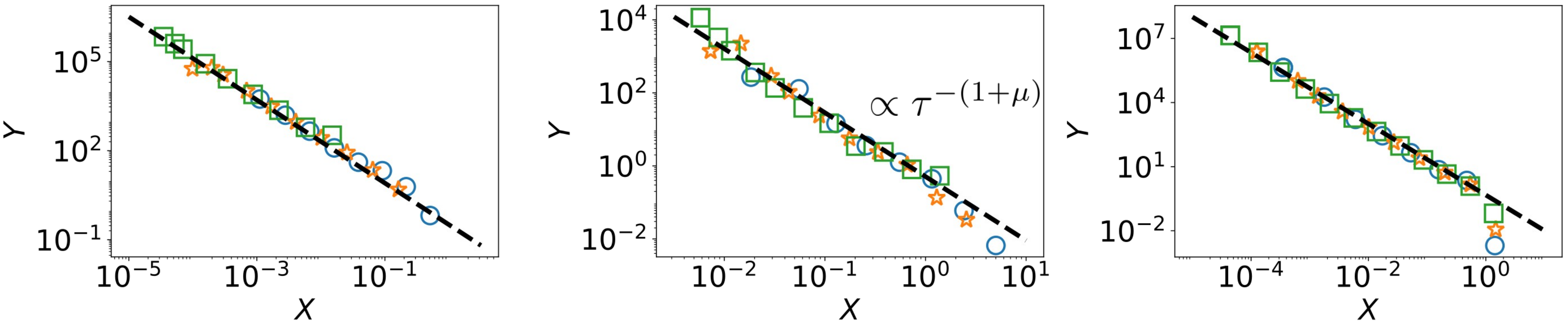
	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
$\mu < 1$ [recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)}$		
$\mu = 1$ [marginal]	$\sqrt{n}$	$n^{3/2}$		$\exp\left[-\text{const}\,(\tau/t_n)^{\mu/(1+\mu)}\right]$	$\exp\left[-\text{const}\,\tau/n^{1/\mu}\right]$
$\mu > 1$ [transient]	1	$n^{(\mu+1)/\mu}$			

Complete characterisation of the time elapsed between visits for random walks:

- In any dimension (3d, ...)
- on disordered media (Percolation clusters, ...)
- on fractals (Sierpinski lattice, ...)
- with anomalous diffusion (Lévy flights, ...)

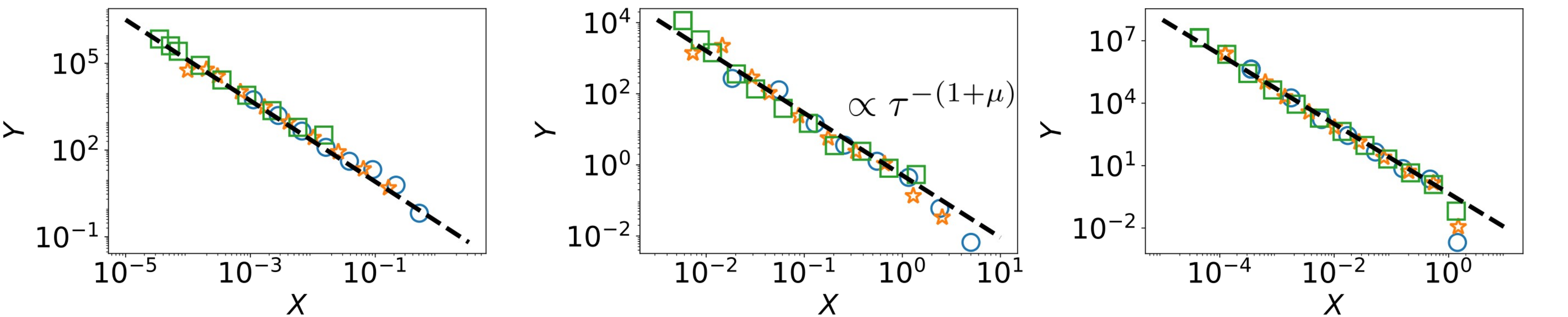
SCAN ME

Openings: Universality?



1d fBm subdiffusive | 1d fBm superdiffusive | 1d True Self-Avoiding RW

$$X = \tau / t_n,\ Y = F_n(\tau) t_n^{\mu+1}$$



1d fBm subdiffusive | 1d fBm superdiffusive | 1d True Self-Avoiding RW

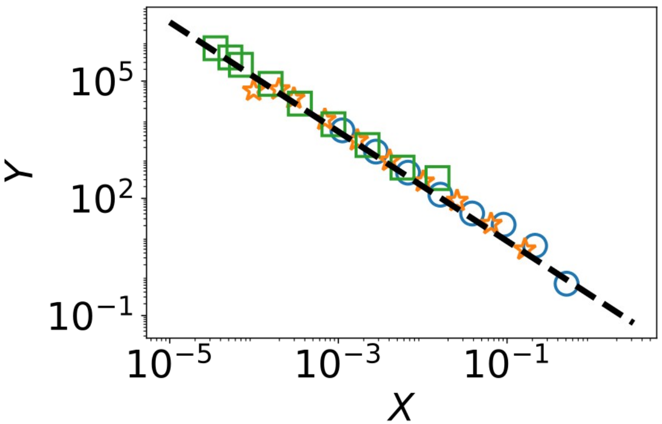
$$X = \tau / t_n, \; Y = F_n(\tau) t_n^{\mu+1}$$

Non-Markovian recurrent RW BUT still:

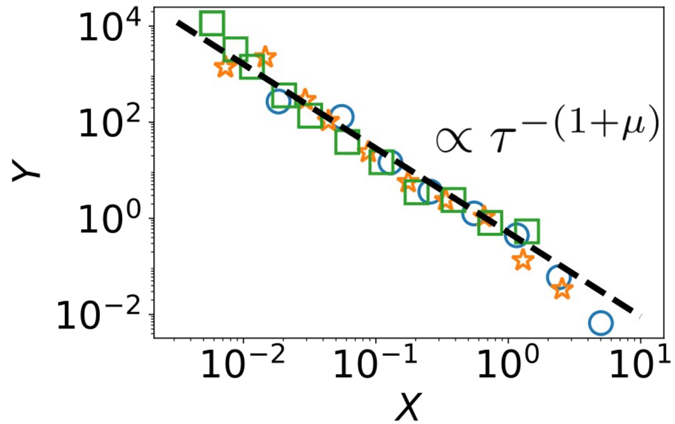
Algebraic

Exponential

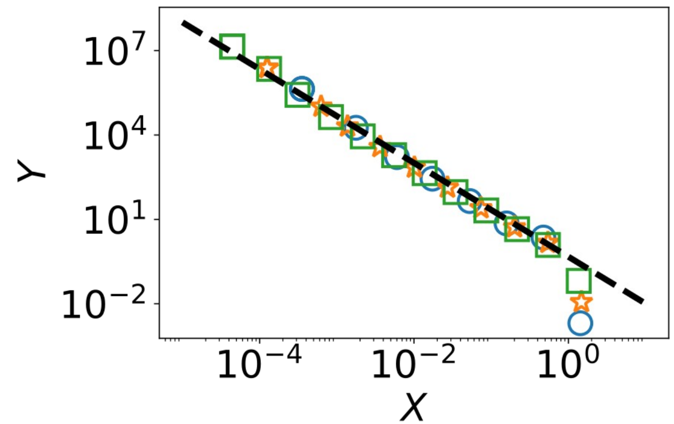
Scale invariance



1d fBm subdiffusive



1d fBm superdiffusive



1d True Self-Avoiding RW

$$X = \tau/t_n,\ Y = F_n(\tau)t_n^{\mu+1}$$

Non-Markovian recurrent RW BUT still:

Algebraic

Exponential

Scale invariance

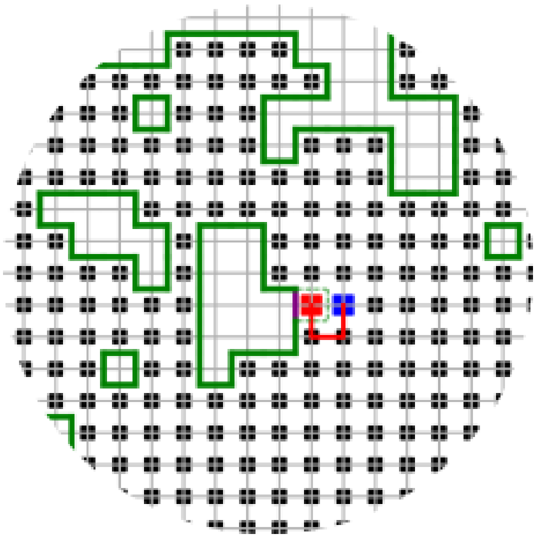
Record ages of non Markovian RWs:

SCAN ME

Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_{\mathrm{w}}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$



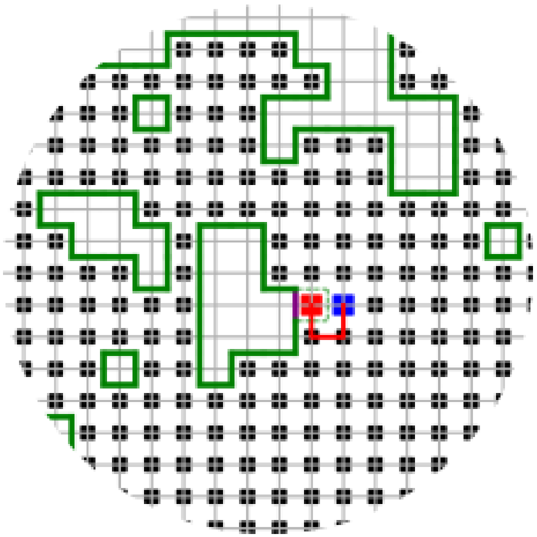
Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_\mathrm{w}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_\mathrm{f} - d_\mathrm{w}$$

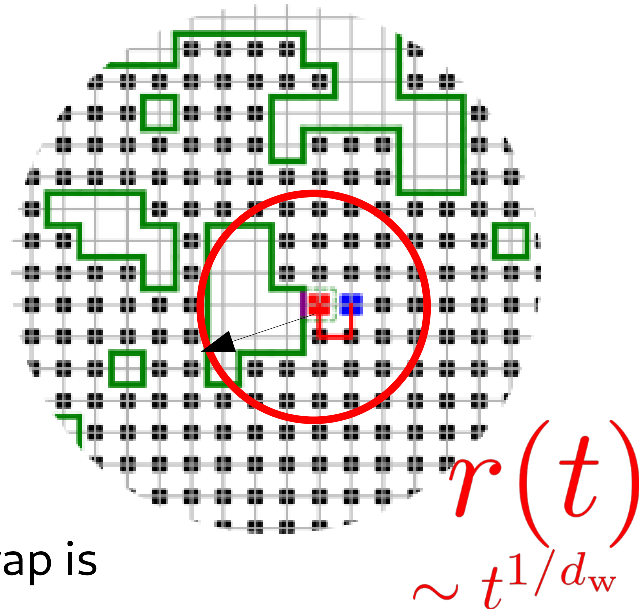


METHOD: $\tau \ll t_n = \rho_n^{d_{\mathrm{w}}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_{\mathrm{f}} - d_{\mathrm{w}}$$



Starting from the last visited site, the probability P_{trap}(t) to be at a trap is

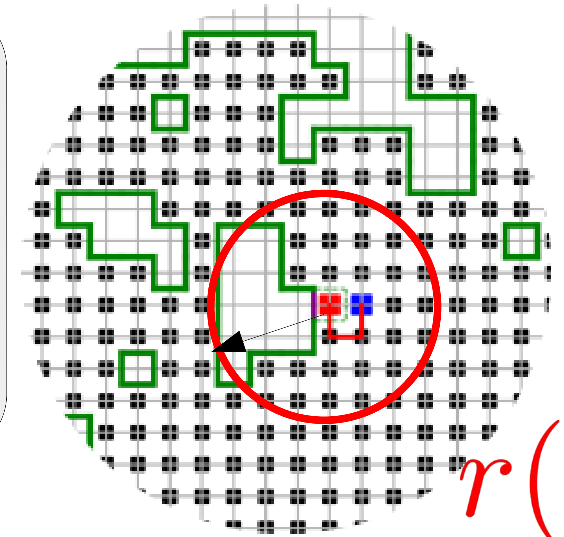
Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_\mathrm{w}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_\mathrm{f} - d_\mathrm{w}$$



Starting from the last visited site, the probability $P_{trap}(t)$ to be at a trap is

$$P_\mathrm{trap}(t) \propto \frac{\text{Number of traps}}{\text{Number of sites}} \propto \frac{r(t)^{d_\mathrm{T}}}{r(t)^{d_\mathrm{f}}} \propto t^{\mu-1}$$

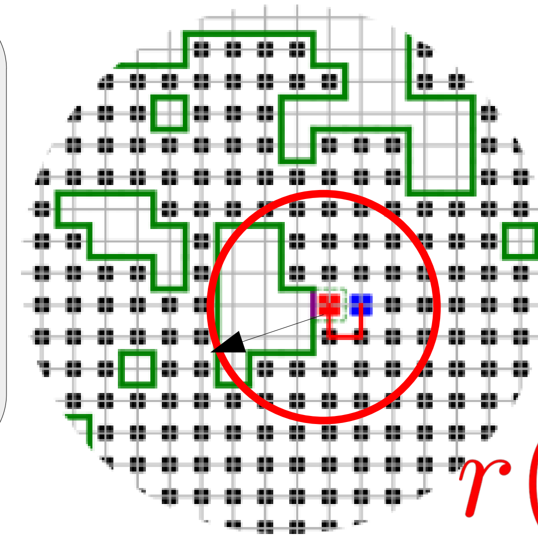
Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_w}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_f - d_w$$



Starting from the last visited site, the probability $P_{trap}(t)$ to be at a trap is

$$P_{\rm trap}(t) \propto \frac{\text{Number of traps}}{\text{Number of sites}} \propto \frac{r(t)^{d_T}}{r(t)^{d_f}} \propto t^{\mu-1}$$

There is a renewal equation between the probability P_{trap} to be at a trap and the probability to first arrive at a trap F_∞,

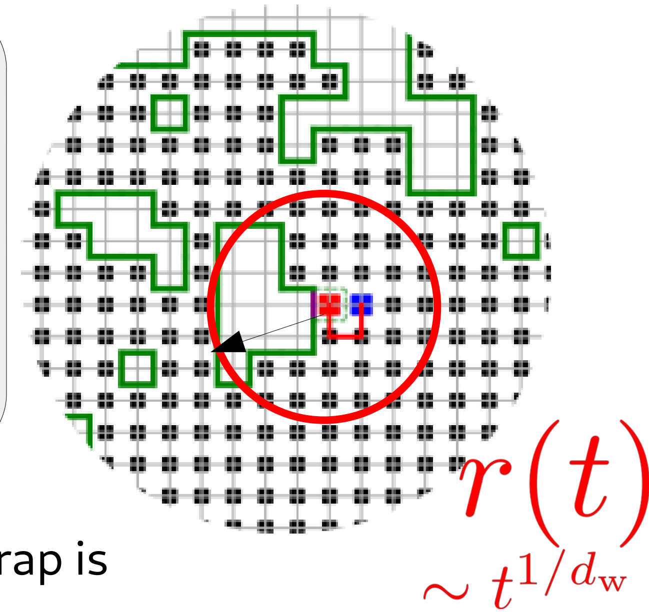
Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_\mathrm{w}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_\mathrm{f} - d_\mathrm{w}$$



Starting from the last visited site, the probability $P_{trap}(t)$ to be at a trap is

$$P_\mathrm{trap}(t) \propto \frac{\text{Number of traps}}{\text{Number of sites}} \propto \frac{r(t)^{d_\mathrm{T}}}{r(t)^{d_\mathrm{f}}} \propto t^{\mu-1}$$

There is a renewal equation between the probability P_{trap} to be at a trap and the probability to first arrive at a trap F_∞,

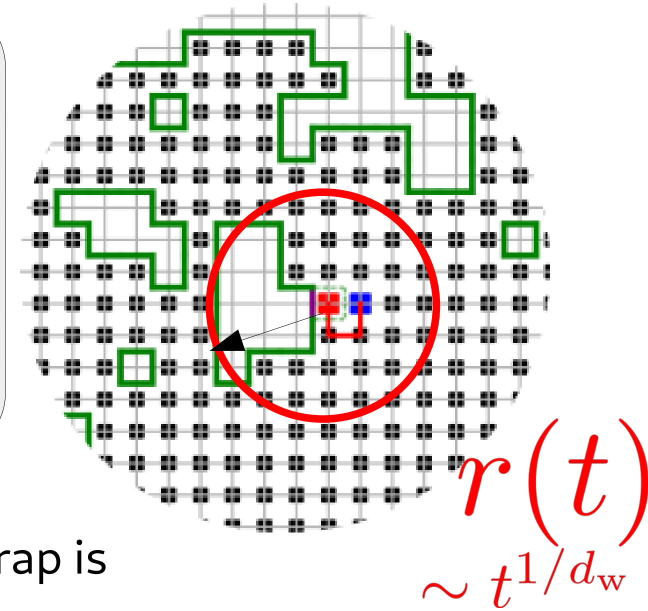
$$P_\mathrm{trap}(t) = \delta(t) + \int_0^t F_\infty(\tau) P_\mathrm{trap}(t-\tau) d\tau$$

METHOD: $\tau \ll t_n = \rho_n^{d_\mathrm{w}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_\mathrm{f} - d_\mathrm{w}$$



Starting from the last visited site, the probability $P_{trap}(t)$ to be at a trap is

$$P_\mathrm{trap}(t) \propto \frac{\text{Number of traps}}{\text{Number of sites}} \propto \frac{r(t)^{d_\mathrm{T}}}{r(t)^{d_\mathrm{f}}} \propto t^{\mu-1}$$

There is a renewal equation between the probability P_{trap} to be at a trap and the probability to first arrive at a trap F_∞,

$$P_\mathrm{trap}(t) = \underbrace{\delta(t)}_{\text{Start close to trap}} + \int_0^t F_\infty(\tau) P_\mathrm{trap}(t-\tau) d\tau$$

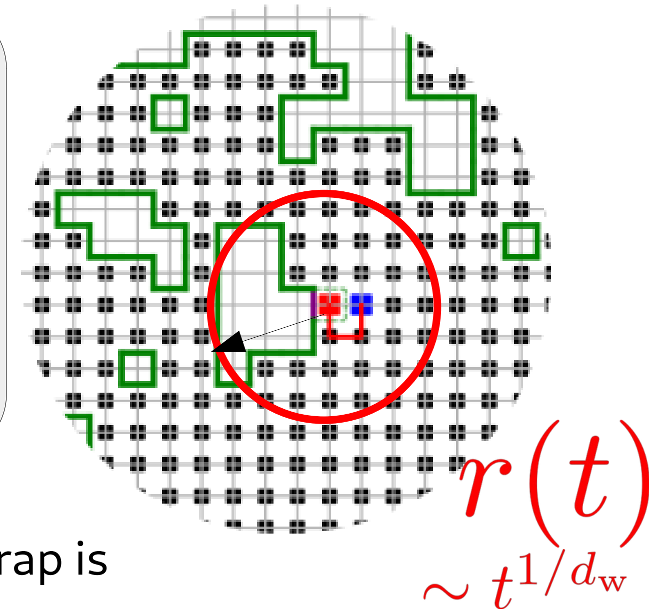
Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_\mathrm{w}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_\mathrm{f} - d_\mathrm{w}$$



Starting from the last visited site, the probability $P_{trap}(t)$ to be at a trap is

$$P_\mathrm{trap}(t) \propto \frac{\text{Number of traps}}{\text{Number of sites}} \propto \frac{r(t)^{d_\mathrm{T}}}{r(t)^{d_\mathrm{f}}} \propto t^{\mu-1}$$

There is a renewal equation between the probability P_{trap} to be at a trap and the probability to first arrive at a trap F_∞,

$$P_\mathrm{trap}(t) = \underbrace{\delta(t)}_{\text{Start close to trap}} + \int_0^t \underbrace{F_\infty(\tau)}_{\text{First arrive at trap}} P_\mathrm{trap}(t-\tau)d\tau$$

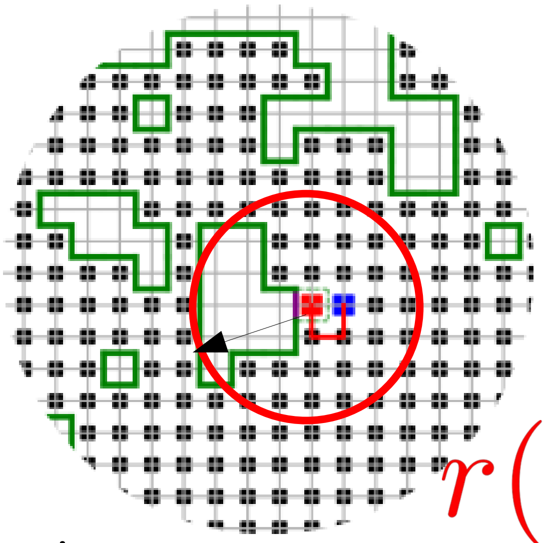
Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_\mathrm{w}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_\mathrm{f} - d_\mathrm{w}$$



Starting from the last visited site, the probability $P_{trap}(t)$ to be at a trap is

$$P_\mathrm{trap}(t) \propto \frac{\text{Number of traps}}{\text{Number of sites}} \propto \frac{r(t)^{d_\mathrm{T}}}{r(t)^{d_\mathrm{f}}} \propto t^{\mu-1}$$

There is a renewal equation between the probability P_{trap} to be at a trap and the probability to first arrive at a trap F_∞,

$$P_\mathrm{trap}(t) = \underbrace{\delta(t)}_{\text{Start close to trap}} + \int_0^t \underbrace{F_\infty(\tau)}_{\text{First arrive at trap}} \underbrace{P_\mathrm{trap}(t-\tau)}_{\text{Start at new trap}} d\tau$$

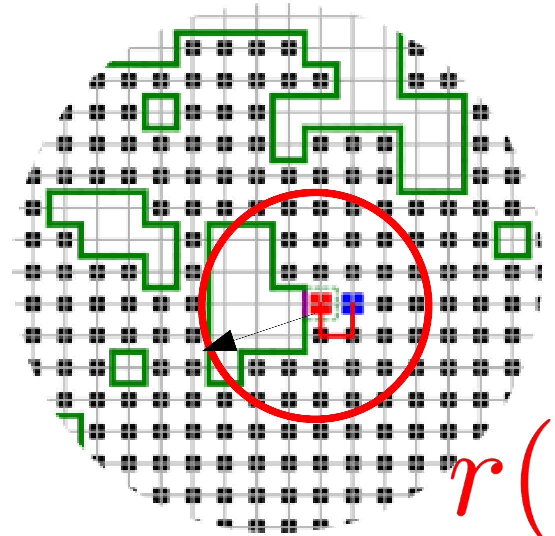
Distribution F_n of the time between visits: early times

METHOD: $\tau \ll t_n = \rho_n^{d_\mathrm{w}}$

The RW sees an infinite visited domain, $F_n \approx F_\infty$

The surface of the visited domain is fractal of dimension

$$d_T = 2d_\mathrm{f} - d_\mathrm{w}$$



Starting from the last visited site, the probability P_{trap}(t) to be at a trap is

$$P_\mathrm{trap}(t) \propto \frac{\text{Number of traps}}{\text{Number of sites}} \propto \frac{r(t)^{d_\mathrm{T}}}{r(t)^{d_\mathrm{f}}} \propto t^{\mu-1}$$

There is a renewal equation between the probability P_{trap} to be at a trap and the probability to first arrive at a trap F_∞,

$$P_\mathrm{trap}(t) = \underbrace{\delta(t)}_{\text{Start close to trap}} + \int_0^t \underbrace{F_\infty(\tau)}_{\text{First arrive at trap}} \underbrace{P_\mathrm{trap}(t-\tau)}_{\text{Start at new trap}} d\tau$$

Algebraic decay, $F_\infty(\tau) \propto \tau^{-(1+\mu)}$

Distribution F_n of the time between visits: intermediate/long times

METHOD: $\tau \gg t_n$

Lower bound for survival probability $S_n(\tau)$ [same as classical trapping problem]

Using the result on $Q_n(r)$ which gives the controlling regions radius at large times,

Distribution F_n of the time between visits: intermediate/long times

METHOD: $\tau \gg t_n$

Lower bound for survival probability $S_n(\tau)$ [same as classical trapping problem]

Using the result on $Q_n(r)$ which gives the controlling regions radius at large times,

$$S_n(\tau) \geq q_n \times \int_0^{n^{1/d_\mathrm{f}}} \frac{1}{\rho_n} \exp\left[-a(r/\rho_n)^{d_\mathrm{f}}\right] \times \exp\left[-b\tau/r^{d_\mathrm{w}}\right] dr$$

Distribution F_n of the time between visits: intermediate/long times

METHOD: $\tau \gg t_n$

Lower bound for survival probability $S_n(\tau)$ [same as classical trapping problem]

Using the result on $Q_n(r)$ which gives the controlling regions radius at large times,

$$S_n(\tau) \geq \underbrace{q_n}_{\mathbb{P}(\text{RW survived up to } \rho_n^{d_\mathrm{w}})} \times \int_0^{n^{1/d_\mathrm{f}}} \frac{1}{\rho_n} \exp\left[-a(r/\rho_n)^{d_\mathrm{f}}\right] \times \exp\left[-b\tau/r^{d_\mathrm{w}}\right] dr$$

Distribution F_n of the time between visits: intermediate/long times

METHOD: $\tau \gg t_n$

Lower bound for survival probability $S_n(\tau)$ [same as classical trapping problem]

Using the result on $Q_n(r)$ which gives the controlling regions radius at large times,

$$S_n(\tau) \geq \underbrace{q_n}_{\mathbb{P}(\text{RW survived up to } \rho_n^{d_w})} \times \int_0^{n^{1/d_f}} \overbrace{\frac{1}{\rho_n} \exp\left[-a(r/\rho_n)^{d_f}\right]}^{Q_n(r)} \times \exp\left[-b\tau/r^{d_w}\right] dr$$

METHOD: $\tau \gg t_n$

Lower bound for survival probability $S_n(\tau)$ [same as classical trapping problem]

Using the result on $Q_n(r)$ which gives the controlling regions radius at large times,

$$S_n(\tau) \geq \underbrace{q_n}_{\mathbb{P}(\text{RW survived up to } \rho_n^{d_w})} \times \int_0^{n^{1/d_f}} \overbrace{\frac{1}{\rho_n} \exp\left[-a(r/\rho_n)^{d_f}\right]}^{Q_n(r)} \times \underbrace{\exp\left[-b\tau/r^{d_w}\right]}_{\mathbb{P}(\text{RW still in the largest ball})} dr$$

Distribution F_n of the time between visits: intermediate/long times

METHOD: $\tau \gg t_n$

Lower bound for survival probability $S_n(\tau)$ [same as classical trapping problem]

Using the result on $Q_n(r)$ which gives the controlling regions radius at large times,

$$S_n(\tau) \geq \underbrace{q_n}_{\mathbb{P}(\text{RW survived up to } \rho_n^{d_w})} \times \int_0^{n^{1/d_f}} \overbrace{\frac{1}{\rho_n} \exp\left[-a(r/\rho_n)^{d_f}\right]}^{Q_n(r)} \times \underbrace{\exp\left[-b\tau/r^{d_w}\right]}_{\mathbb{P}(\text{RW still in the largest ball})} dr$$

RESULTS:

$S_n(\tau)$ has same time dependence as its lower bound

$$F_n(\tau) = -\frac{dS_n}{d\tau} \propto \exp\left[-\text{const}(\tau/t_n)^{\mu/(1+\mu)}\right]$$

Distribution F_n of the time between visits: intermediate/long times

METHOD: $\tau \gg t_n$

Lower bound for survival probability $S_n(\tau)$ [same as classical trapping problem]

Using the result on $Q_n(r)$ which gives the controlling regions radius at large times,

$$S_n(\tau) \geq \underbrace{q_n}_{\mathbb{P}(\text{RW survived up to } \rho_n^{d_w})} \times \int_0^{n^{1/d_f}} \overbrace{\frac{1}{\rho_n} \exp\left[-a(r/\rho_n)^{d_f}\right]}^{Q_n(r)} \times \underbrace{\exp\left[-b\tau/r^{d_w}\right]}_{\mathbb{P}(\text{RW still in the largest ball})} dr$$

RESULTS:

$S_n(\tau)$ has same time dependence as its lower bound

$$F_n(\tau) = -\frac{dS_n}{d\tau} \propto \exp\left[-\text{const}(\tau/t_n)^{\mu/(1+\mu)}\right]$$

+at long time when statistics is dominated by the upper bound of the integral,

$$F_n(\tau) \propto \exp\left[-\text{const}\ \tau/n^{1/\mu}\right] \qquad \tau \gg T_n$$