Universal exploration dynamics of random walks

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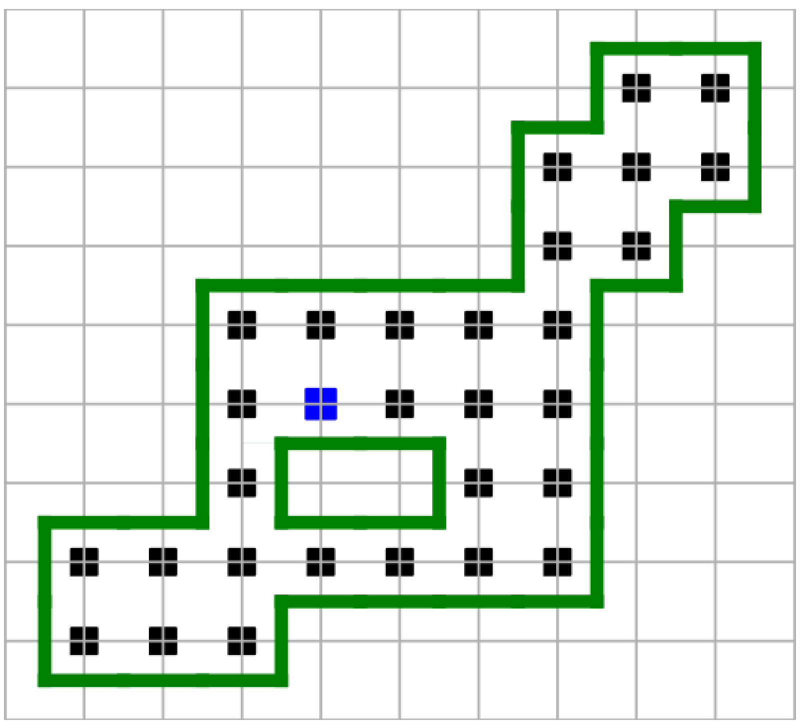
A simple 2d random walker has just visited n sites:
How long does it take to visit a new site?



■ : visited sites.

| : boundary between visited/univisited sites.

■ : nth visited site.

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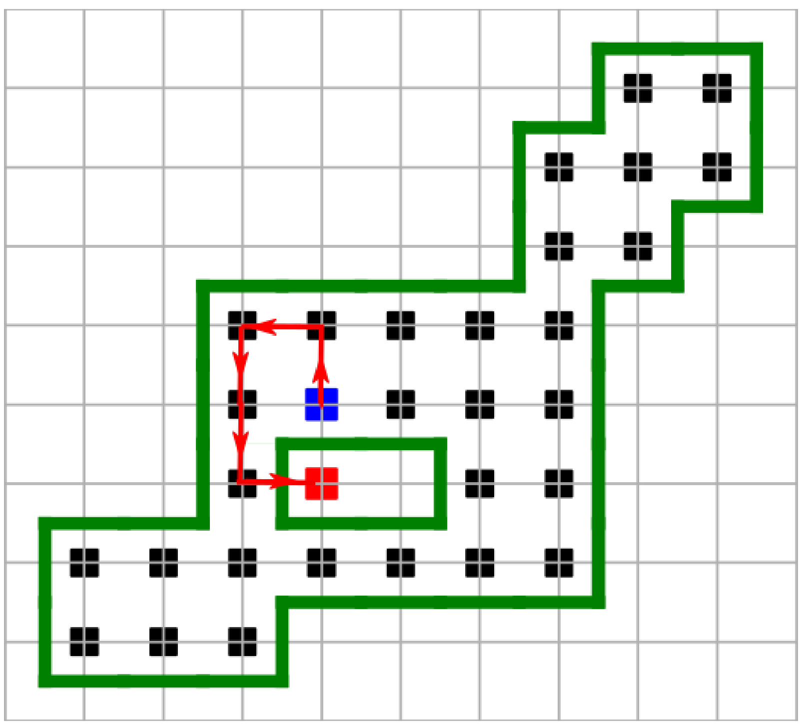


■ : visited sites.

| : boundary between visited/univisited sites.

■ : nth visited site.

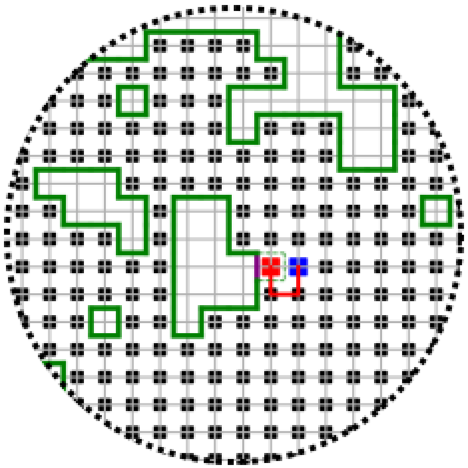
■ : $(n+1)$st visited site.

$\mathcal{T}_n$ = time elapsed between the visit of the nth and the $(n+1)$st new sites [distribution F_n].

(here, n =30 and $\mathcal{T}_n$=5)

Distribution F_n of the time between visits

Early-time algebraic regime



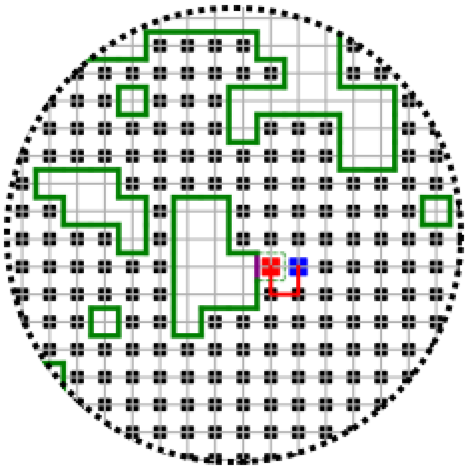
Semi-infinite visited domain
+ Surface and volume of visited domain are fractal:
-> Algebraic decay

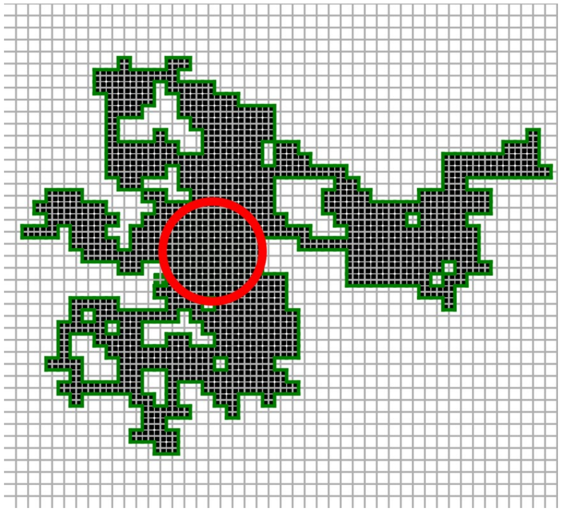
$$F_n(\tau) \propto \tau^{-2}$$

Distribution F_n of the time between visits

Early-time algebraic regime

Intermediate-time stretched exponential regime





Semi-infinite visited domain + Surface and volume of visited domain are fractal:
-> Algebraic decay

Exponential distribution on the volume of largest fully visited ball + exponential exit time of ball:
-> Stretched exponential

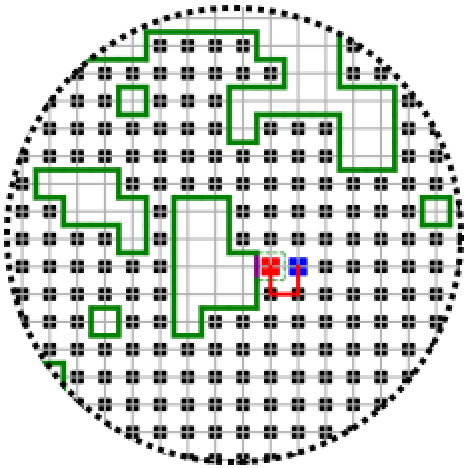
$$t_n = \sqrt{n}$$

$$F_n(\tau) \propto \tau^{-2}$$

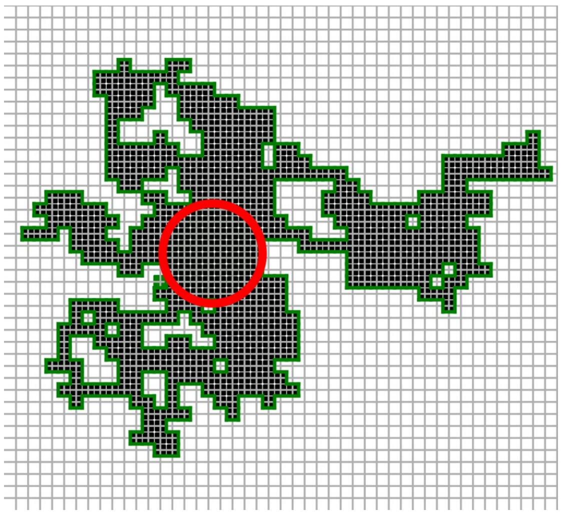
$$\propto \exp\left[-\text{const.}\sqrt{\tau/\sqrt{n}}\right]$$

Distribution F_n of the time between visits

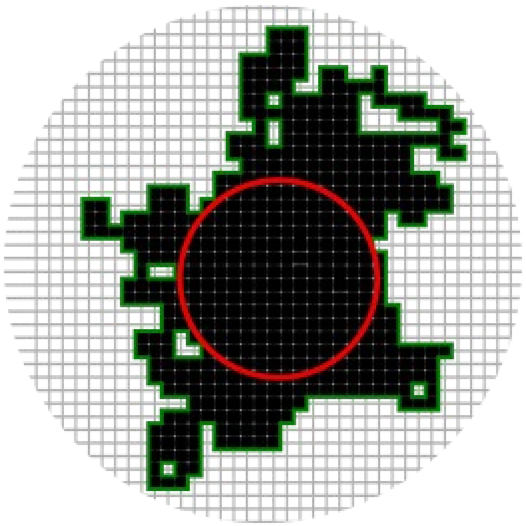
Early-time algebraic regime



Intermediate-time stretched exponential regime



Long-time exponential regime



Semi-infinite visited domain
+ Surface and volume of visited domain are fractal:
-> Algebraic decay

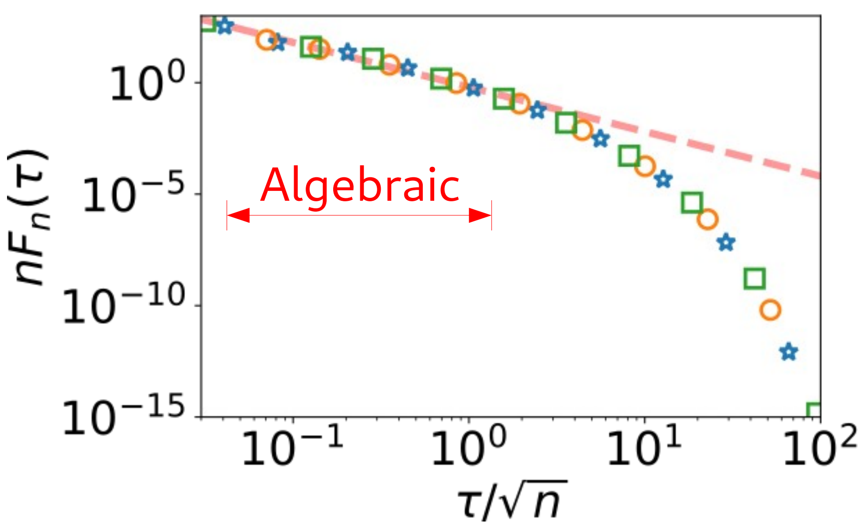
Exponential distribution on the volume of largest fully visited ball + exponential exit time of ball:
-> Stretched exponential

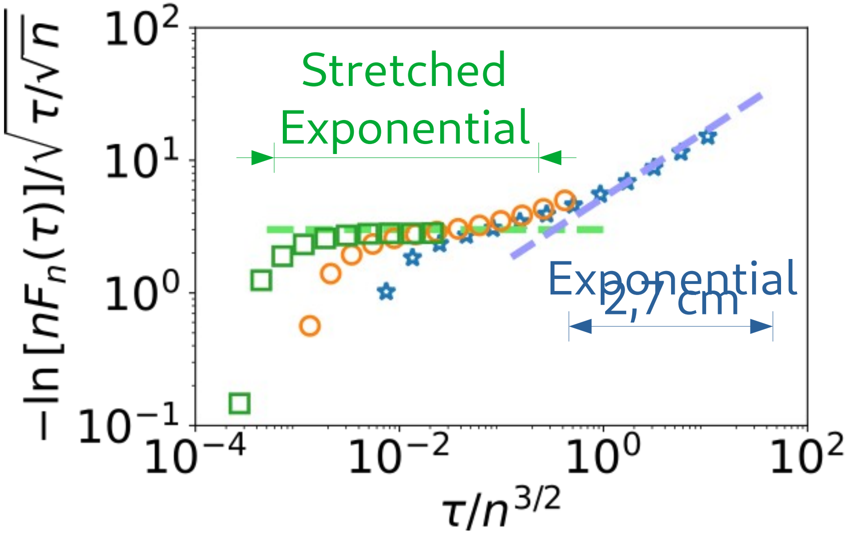
Volume fully visited is maximal:
-> Exponential

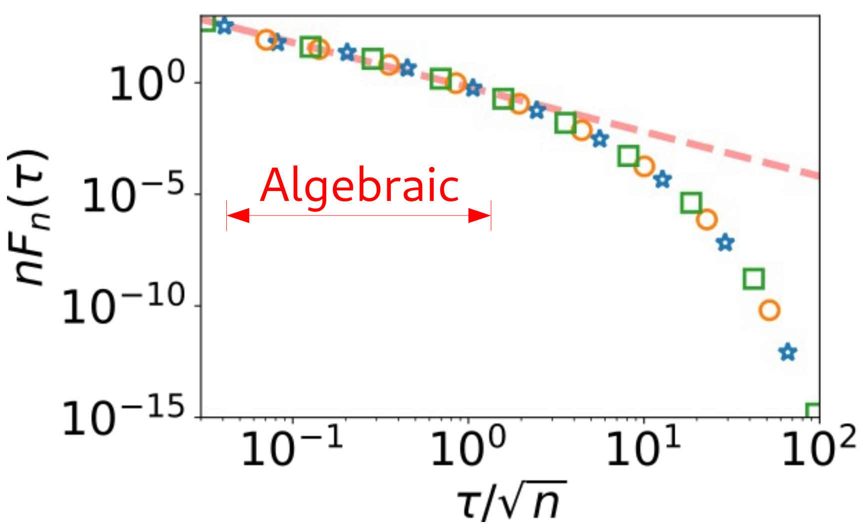
$$t_n = \sqrt{n} \qquad T_n = n^{3/2}$$

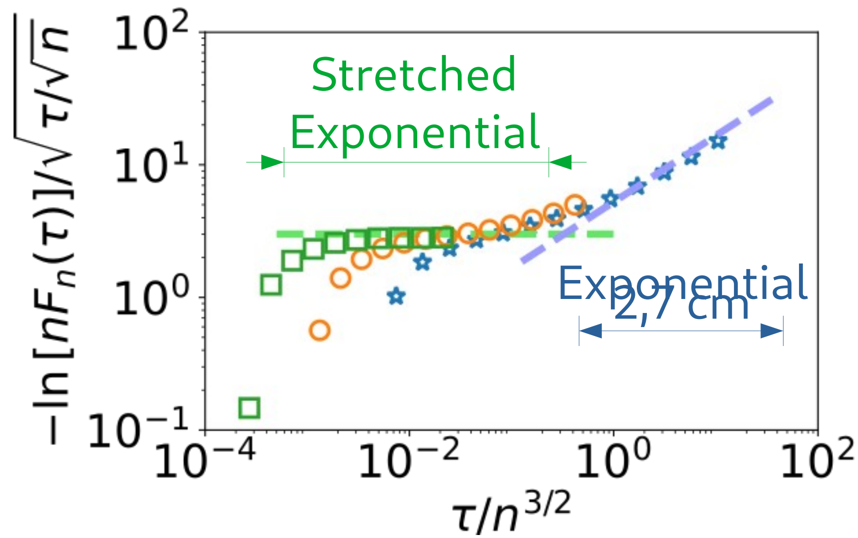
$$F_n(\tau) \propto \tau^{-2} \quad \Big| \quad \propto \exp\left[-\text{const.}\sqrt{\tau/\sqrt{n}}\right] \quad \Big| \quad \propto \exp\left[-\text{const.}\tau/n\right]$$

Numerical check & Universality









Generalisation to any Markovian process in any medium: Hypercubic lattices, Lévy flights, Sierpinski lattice, percolation clusters, persistent random walks (and even some non-Markovian)

	t_n	T_n	$1 \ll \tau \ll t_n$	$t_n \ll \tau \ll T_n$	$T_n \ll \tau$
[recurrent]	$n^{1/\mu}$	$n^{1/\mu}$	$\tau^{-(1+\mu)} \equiv \tau^{-(2-\theta)}$		
[marginal]	$\sqrt{n}$	$n^{3/2}$		$\exp\Big[-\text{const}\,(\tau/t_n)^{\mu/(1+\mu)}\Big]$	$\exp\Big[-\text{const}\,\tau/n^{1/\mu}\Big]$
[transient]	1	$n^{(\mu+1)/\mu}$			