# Record ages of non-Markovian Scale Invariant Random Walks

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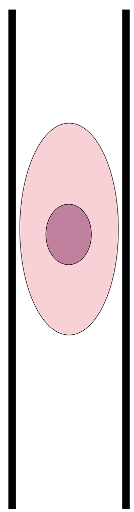
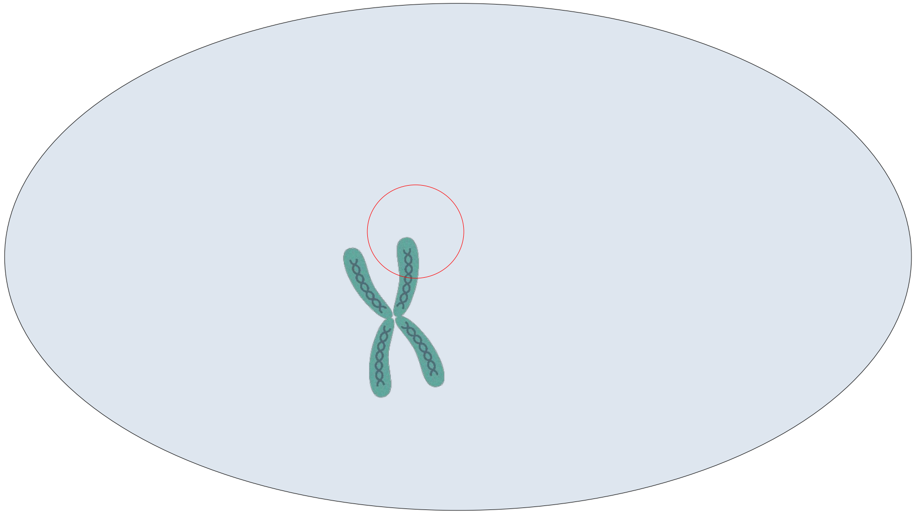


Movement of a MDCK epithelial cell on a 1d track. **(d'Alessandro, *Nat. Comm. 12 4118*, 2021)**



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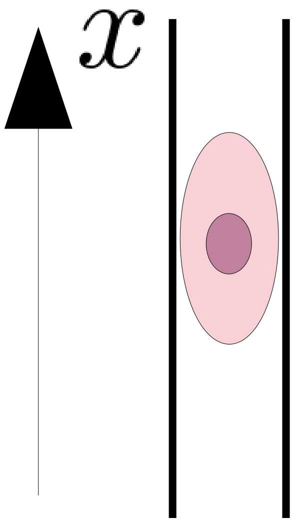


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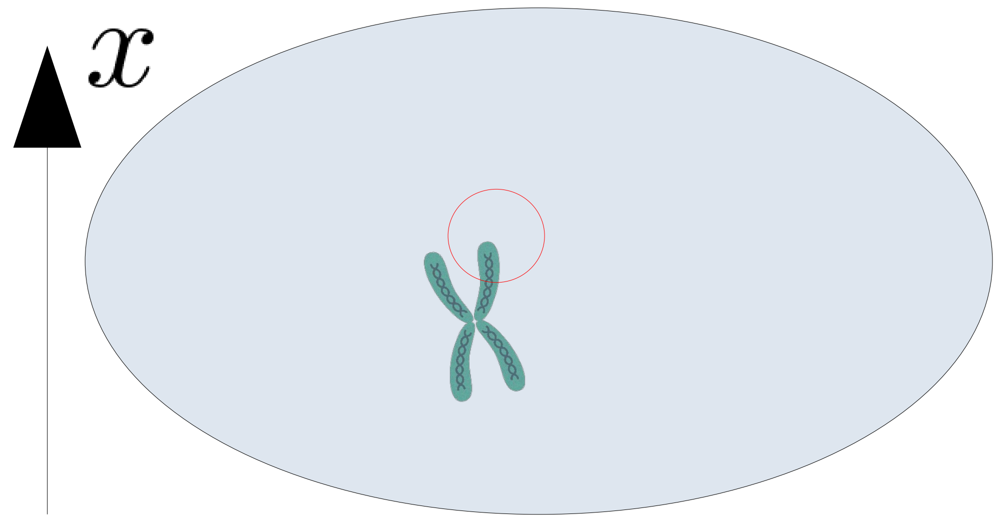


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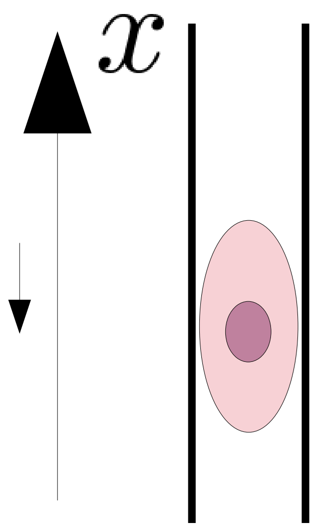


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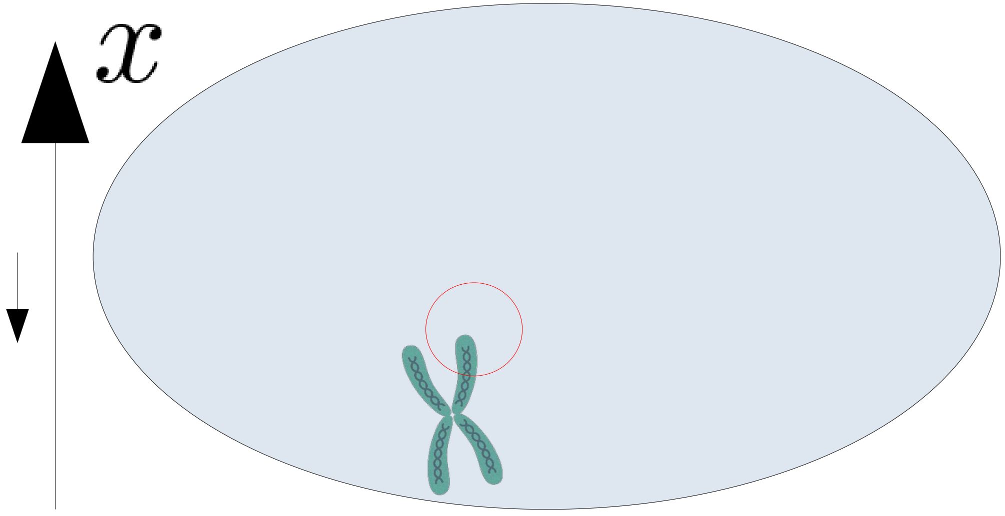


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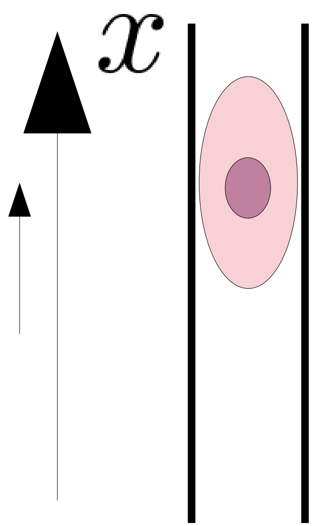


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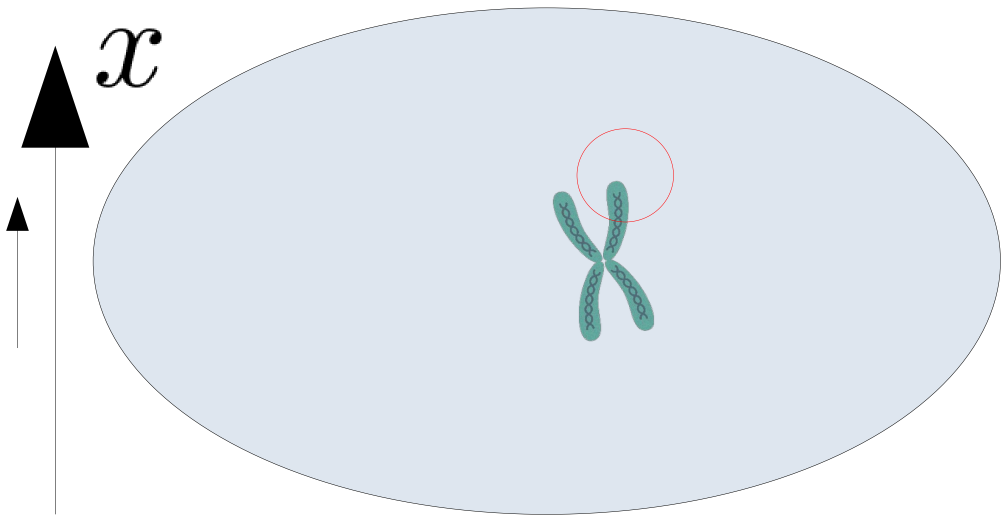


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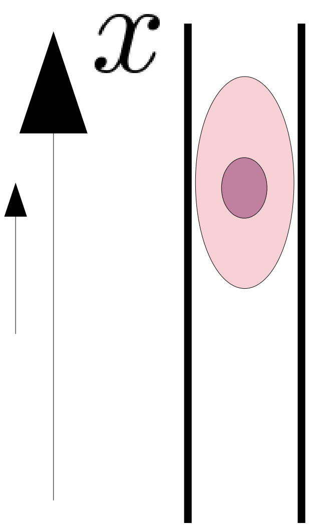


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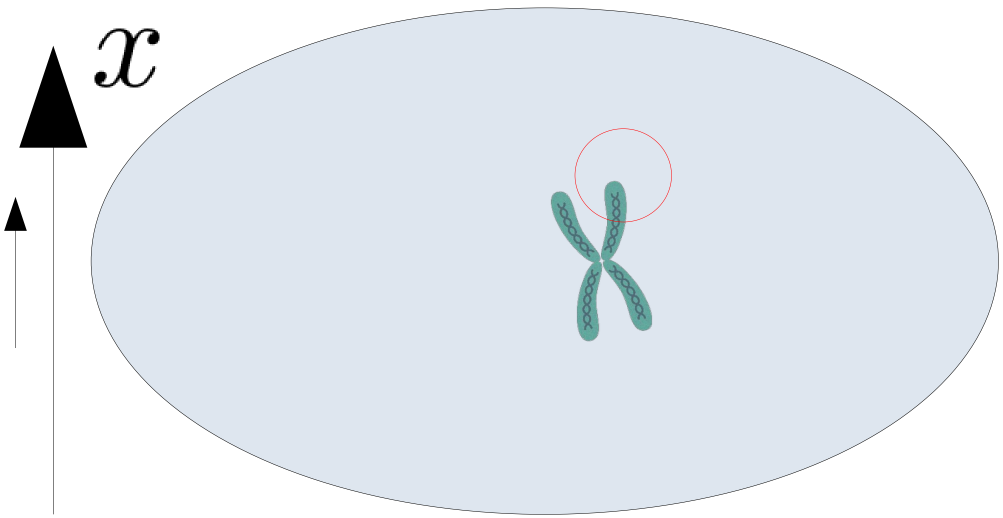


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**Model: Non-Markovian Random Walk (RW)**
Memory: very hard problem.

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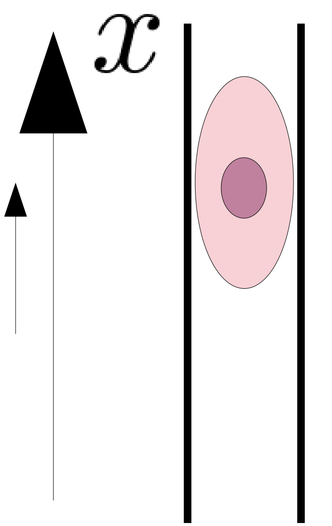


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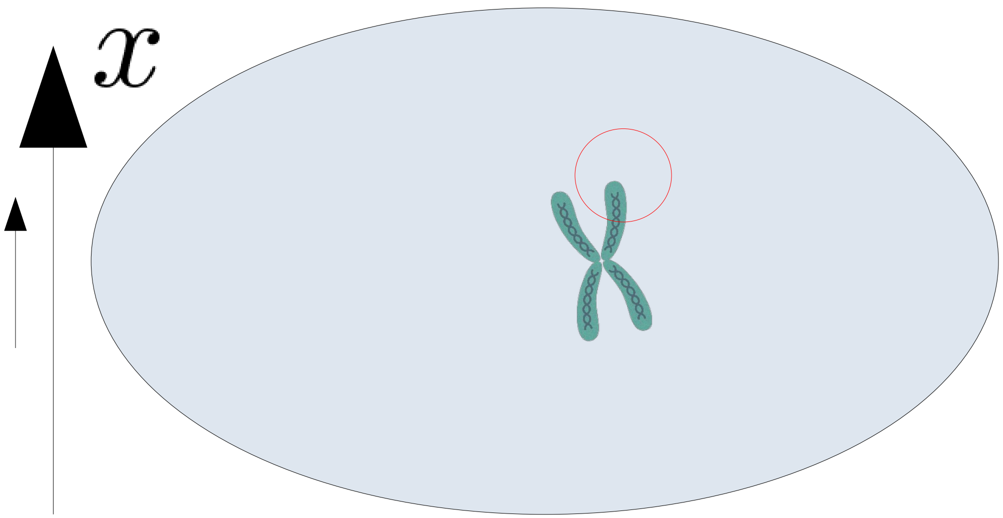


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Memory: very hard problem.
Results independent of details?
First passage observables [records]?

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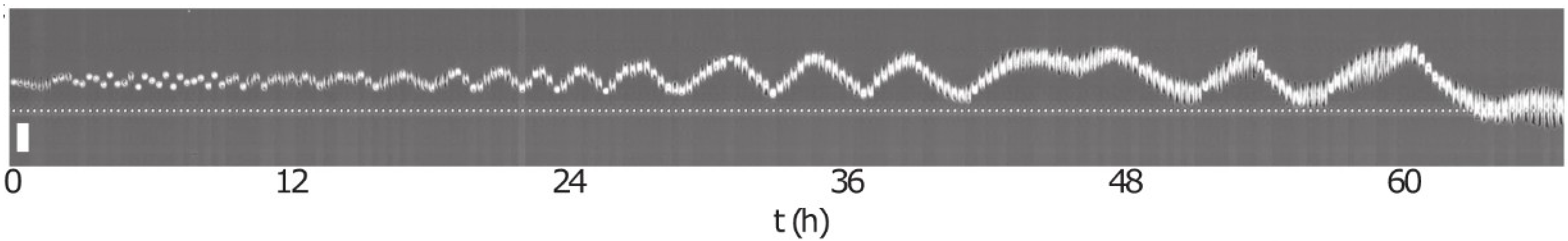


Position of a MDCK epithelial cell
on a 1d track as a function of time. (d'Alessandro '21)

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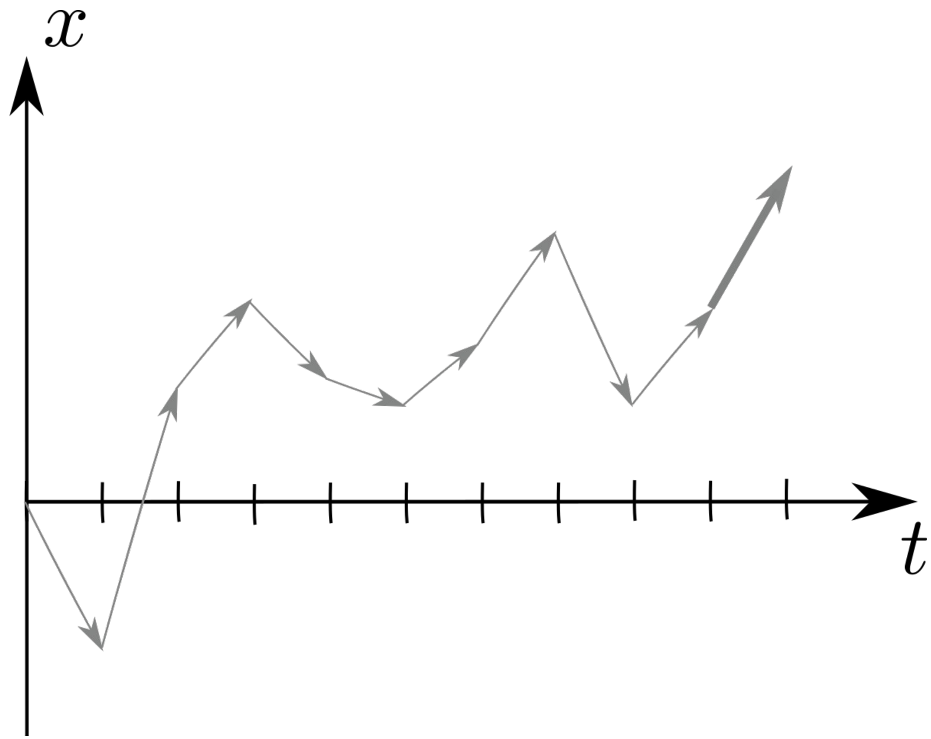

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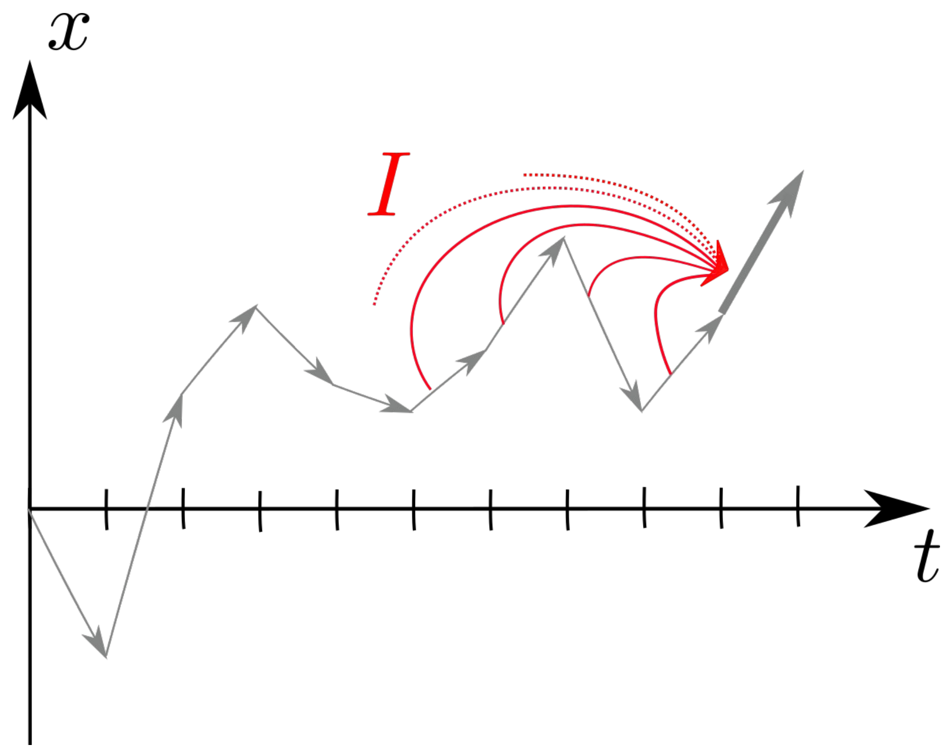


What happens when the RW...

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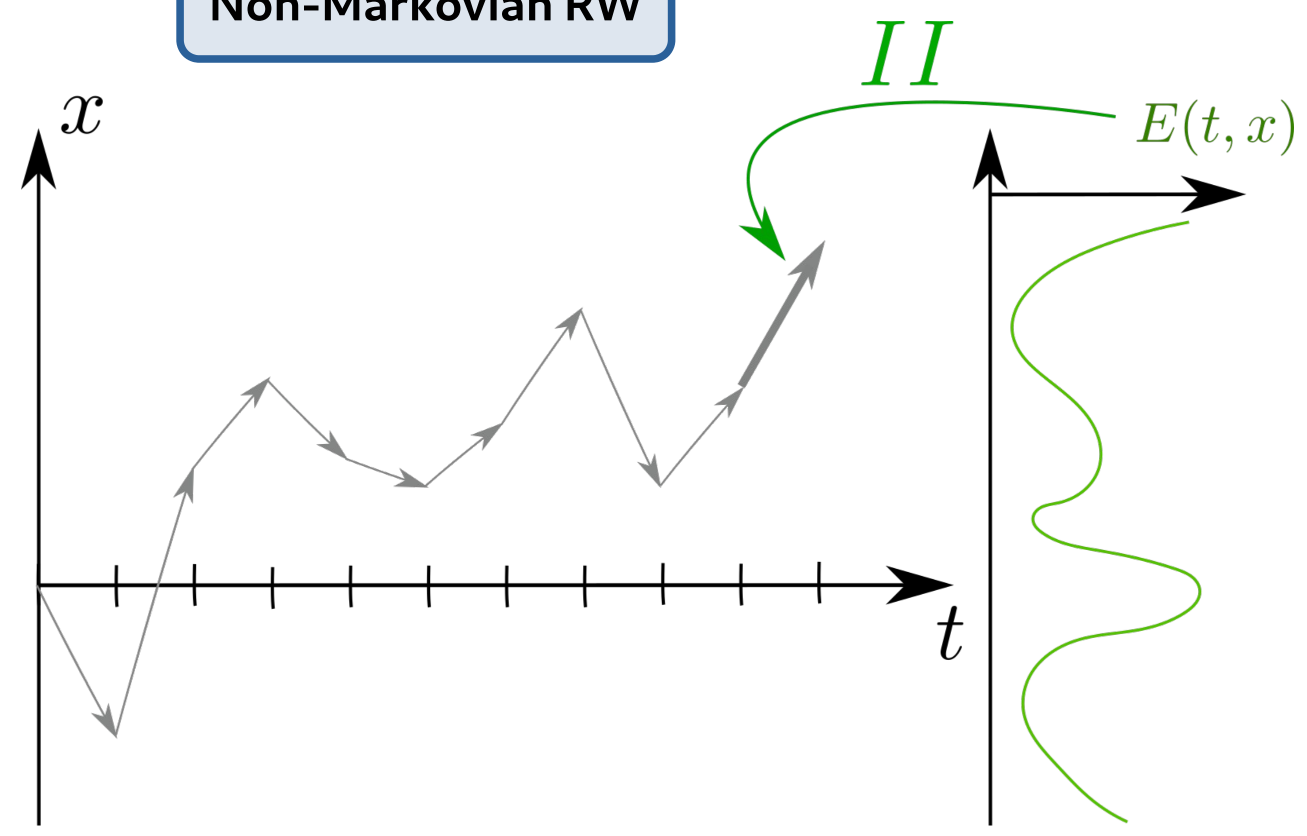


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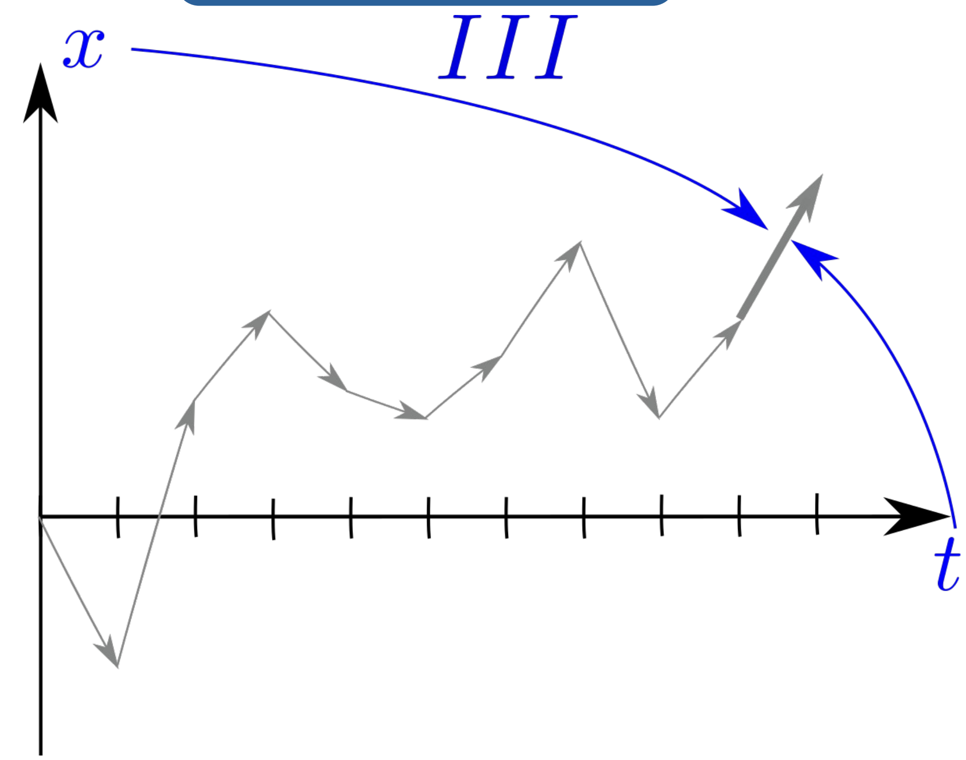


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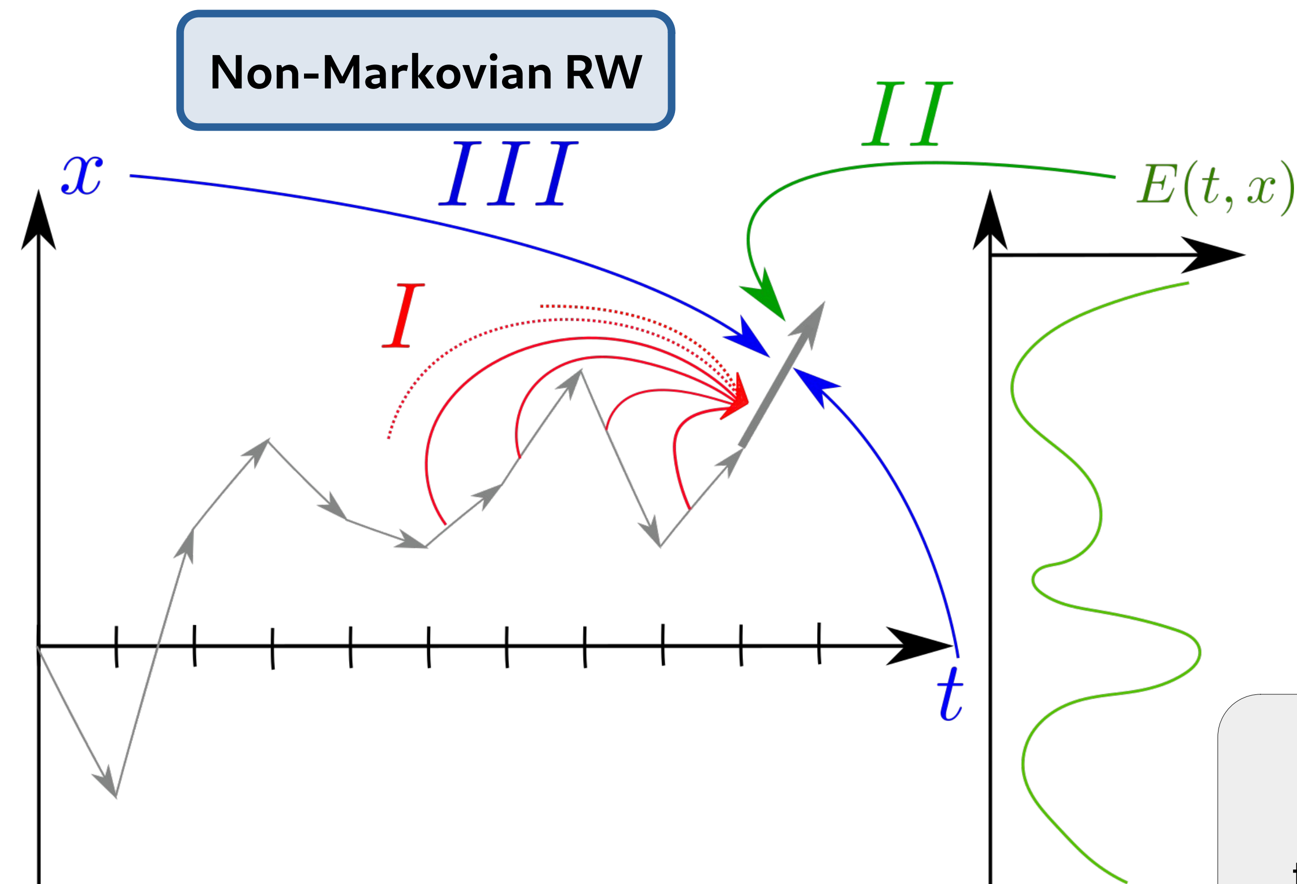


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**“Non-Markovian is the rule”**: River fluxes, volcanic soil temperature, air temperature, cell displacement, Ethernet traffic...

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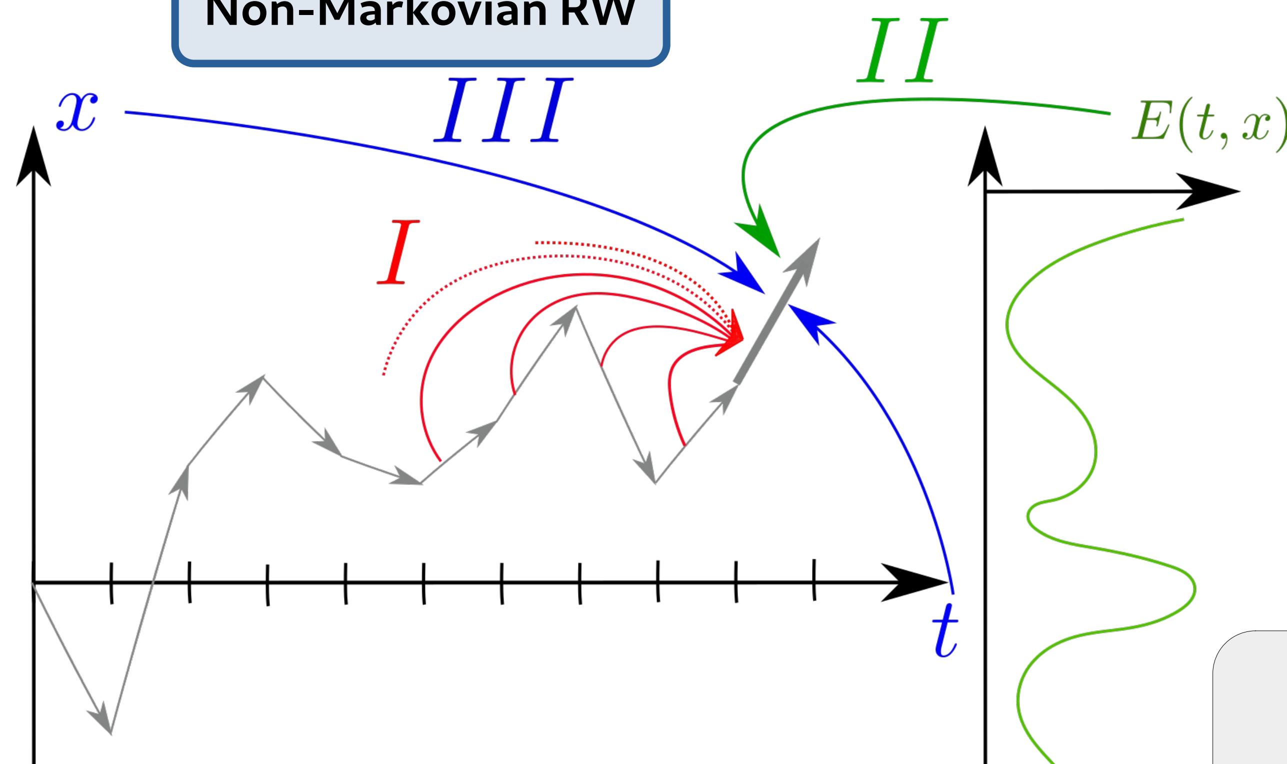


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**OBJECTIVE:** Is there any universal characteristics of these models (regardless of the details)?

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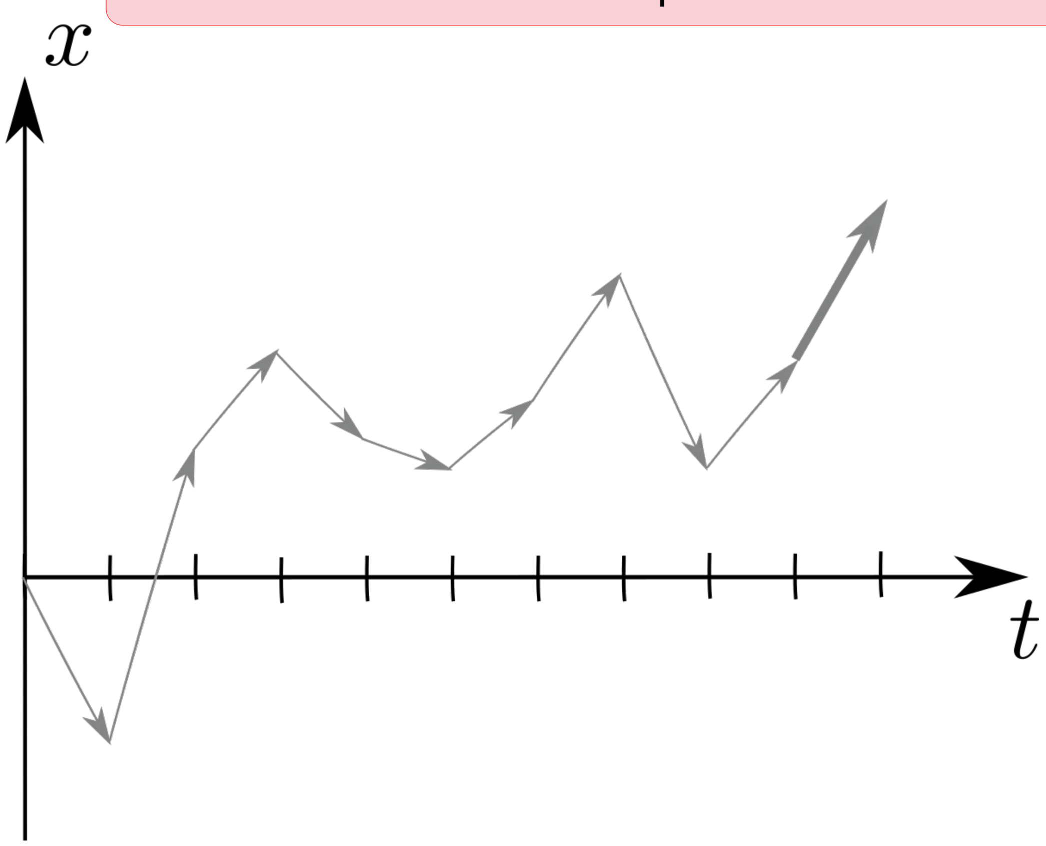
Focus on the extremal positions of the walk



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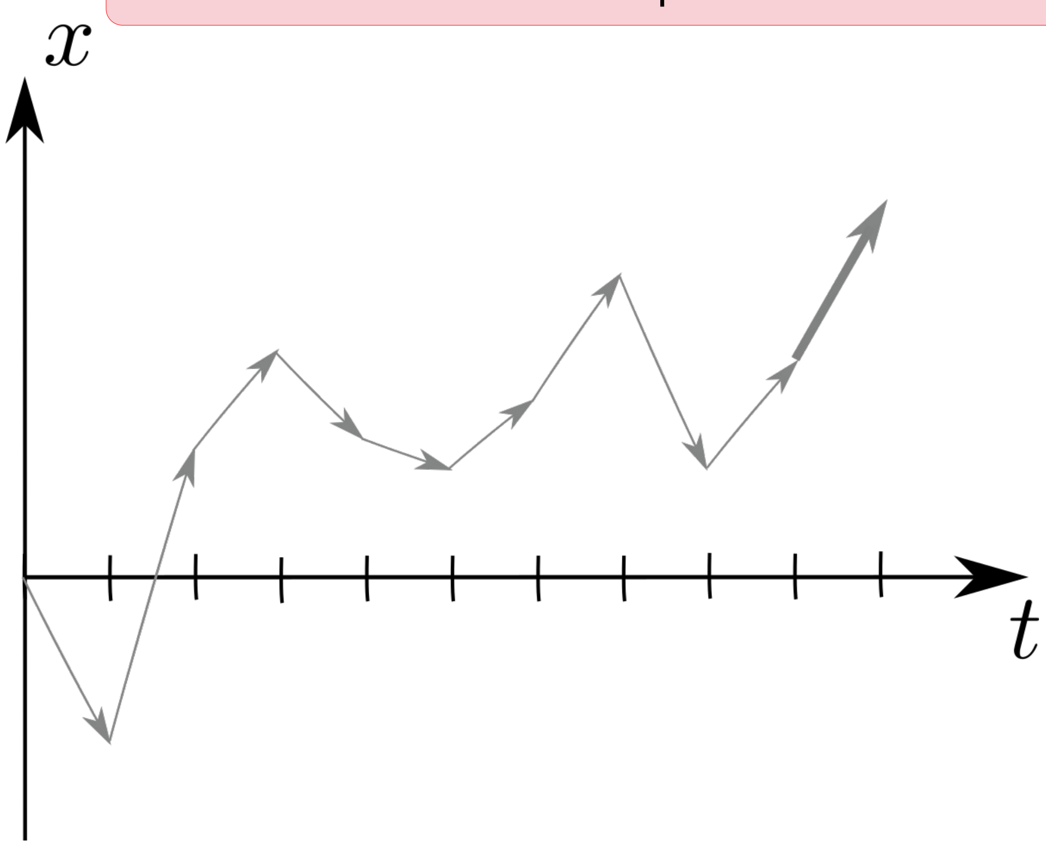


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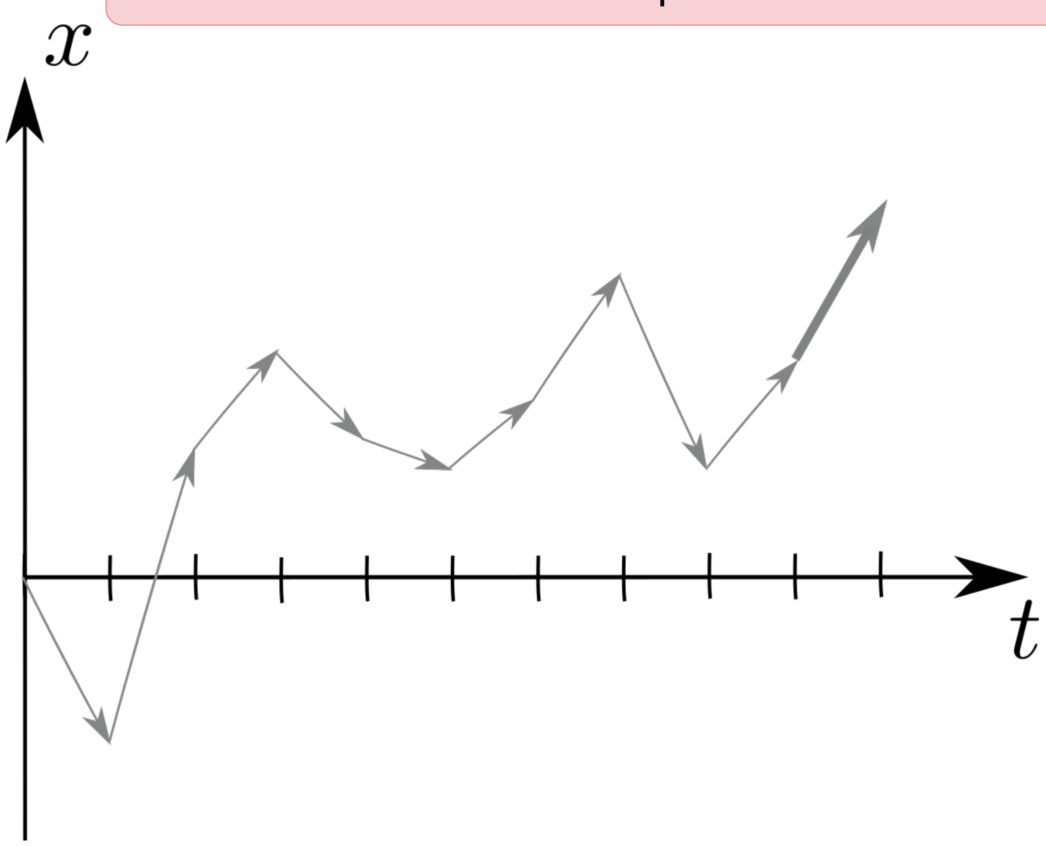


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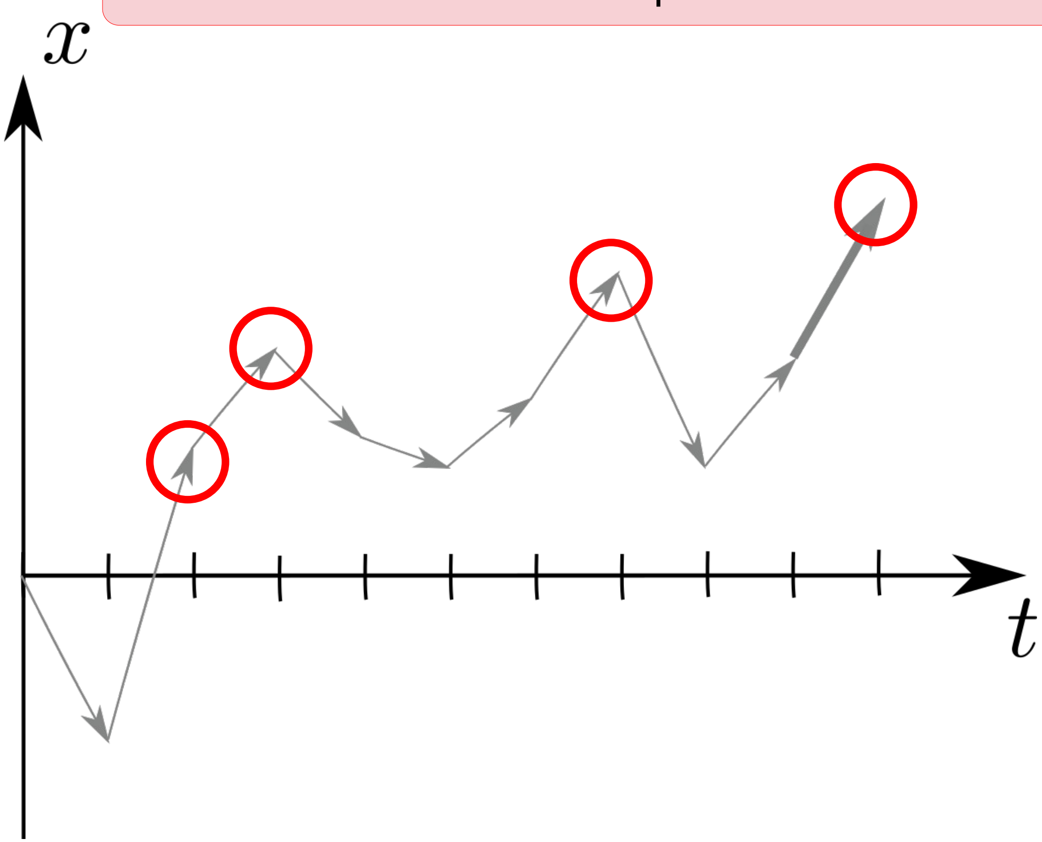


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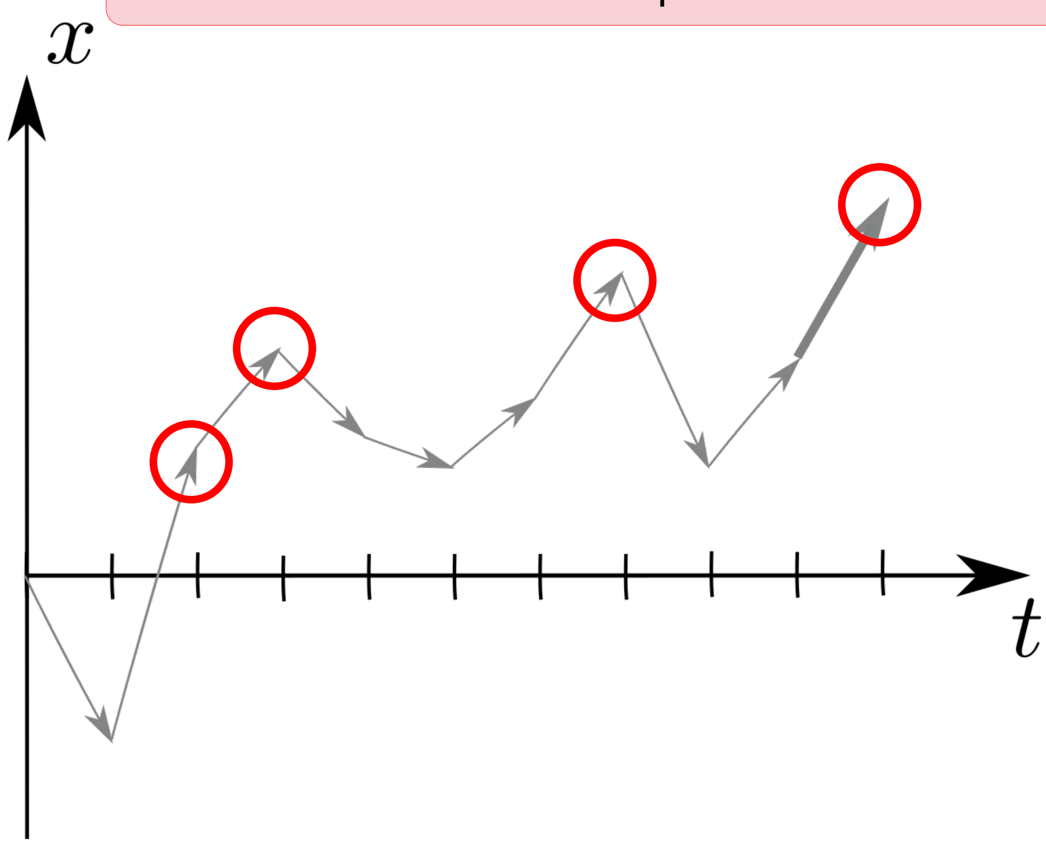


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Time elapsed between the $n^{th}$ and $(n+1)^{st}$ record.

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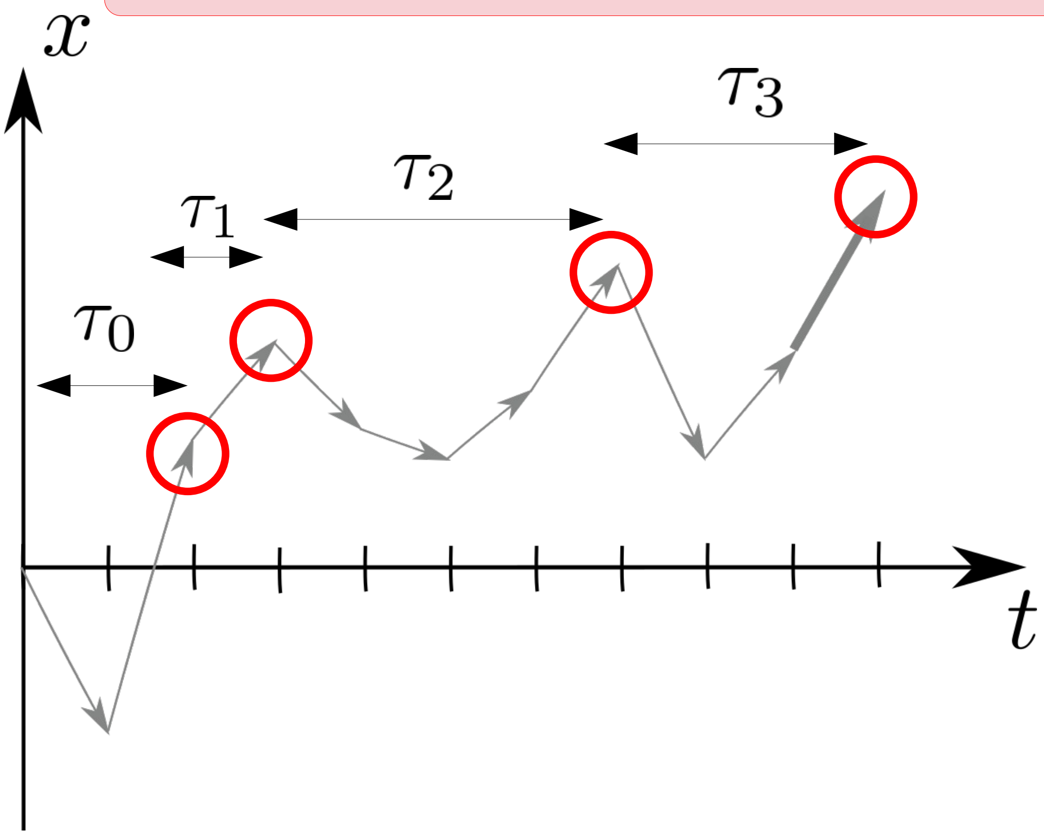


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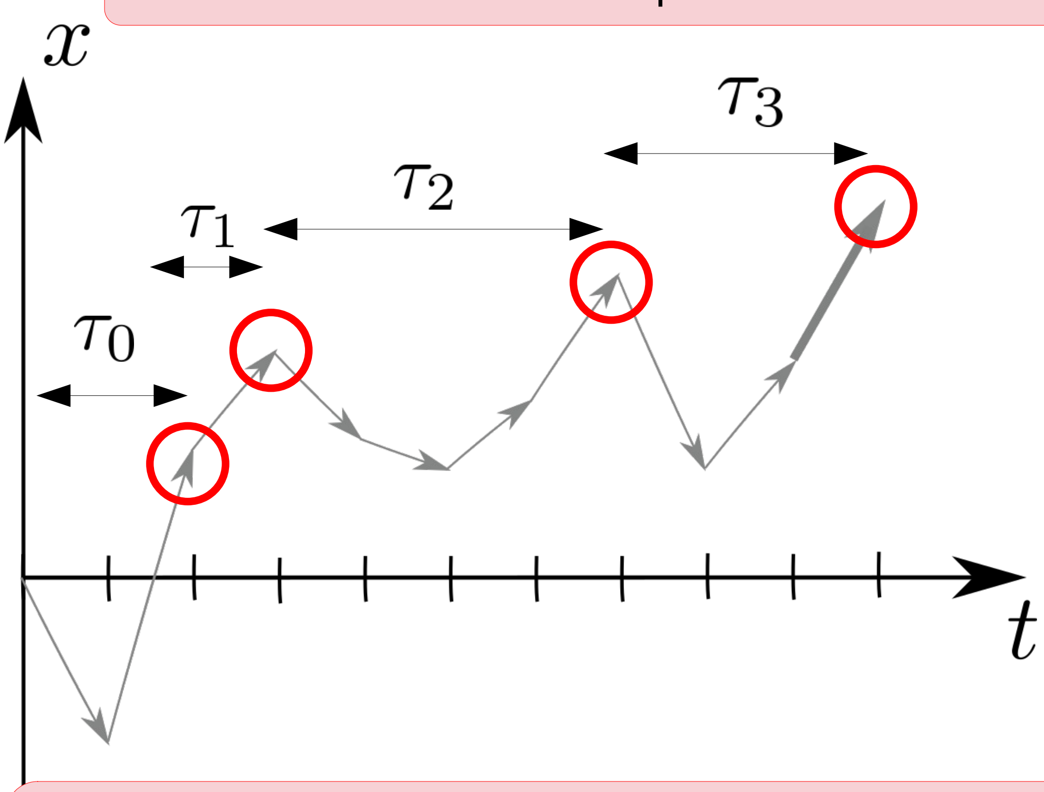


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**OBJECTIVE:** Is there any universal behaviour of the record ages (regardless of the model details)?

# Results in the Markovian case

For a Markovian (memoryless) RW, the distribution of the record age is algebraic of exponent 1/2 and independent of the current number of records.
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It is very different for non-Markovian RWs: memory matters!

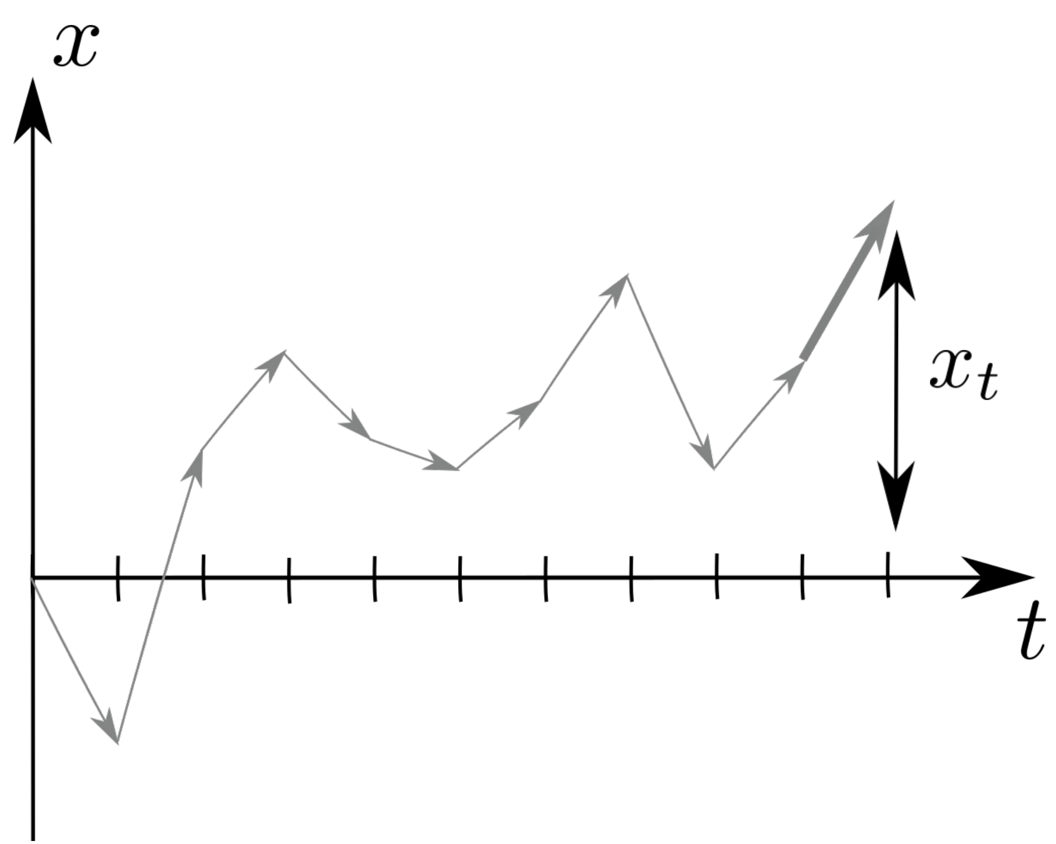
# Definitions

We consider 1d symmetric, non-Markovian, RWs at discrete times, **scale-invariant**

Typical displacement of the RW after time t:

$$x_t \propto t^{1/d_\mathrm{w}}$$

$d_\mathrm{w}$=walk dimension (2 for diffusive systems)



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$$x_{t,T} = x_{T+t} - x_T \propto T^{1/d_{\mathrm{w}} - 1/d_{\mathrm{w}}^0} t^{1/d_{\mathrm{w}}^0}$$

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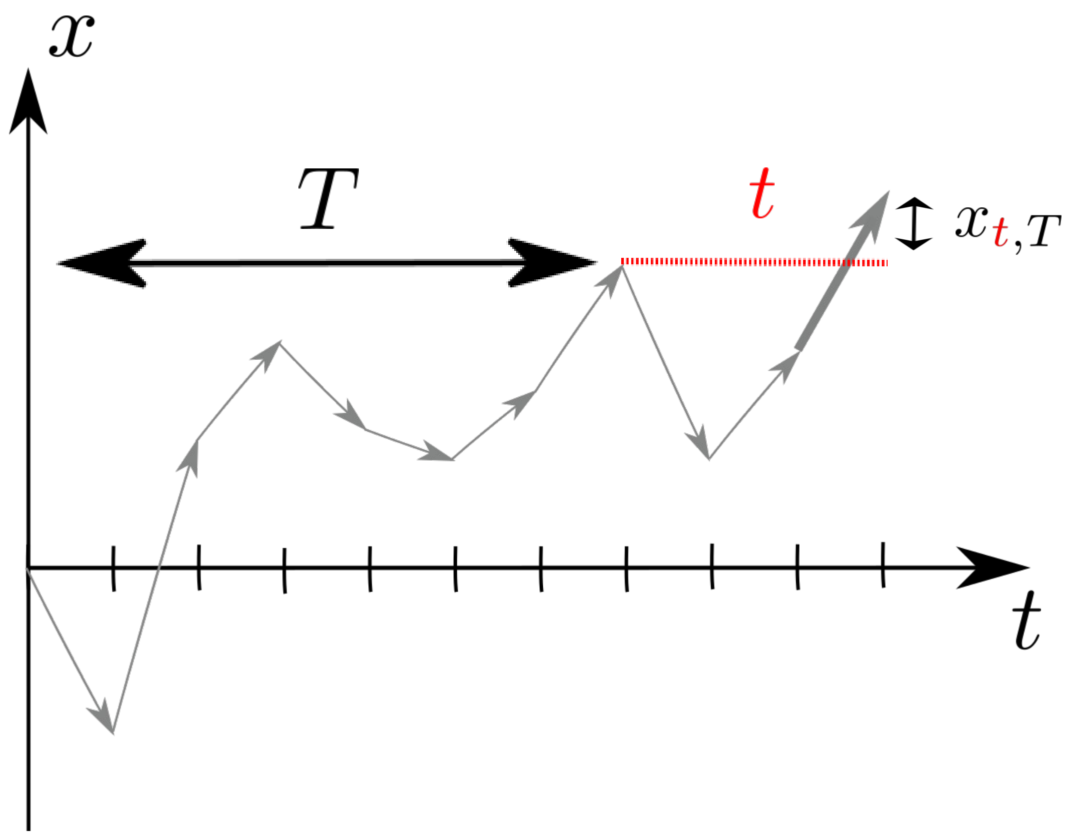

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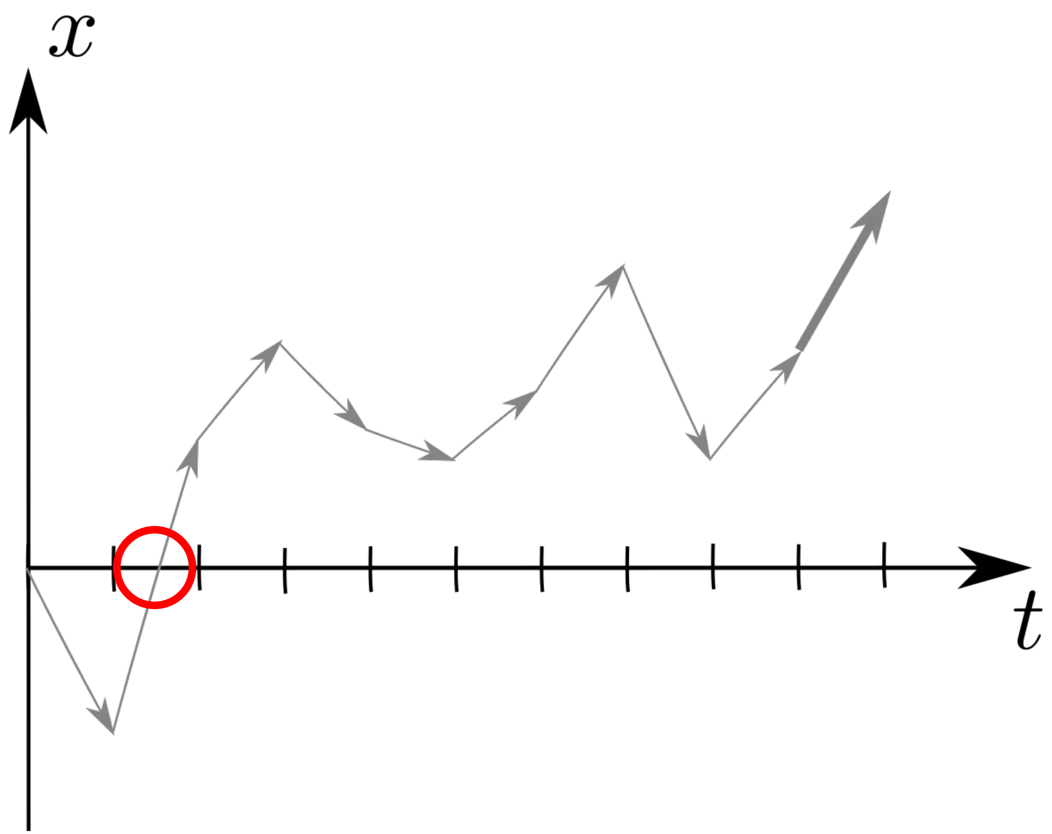

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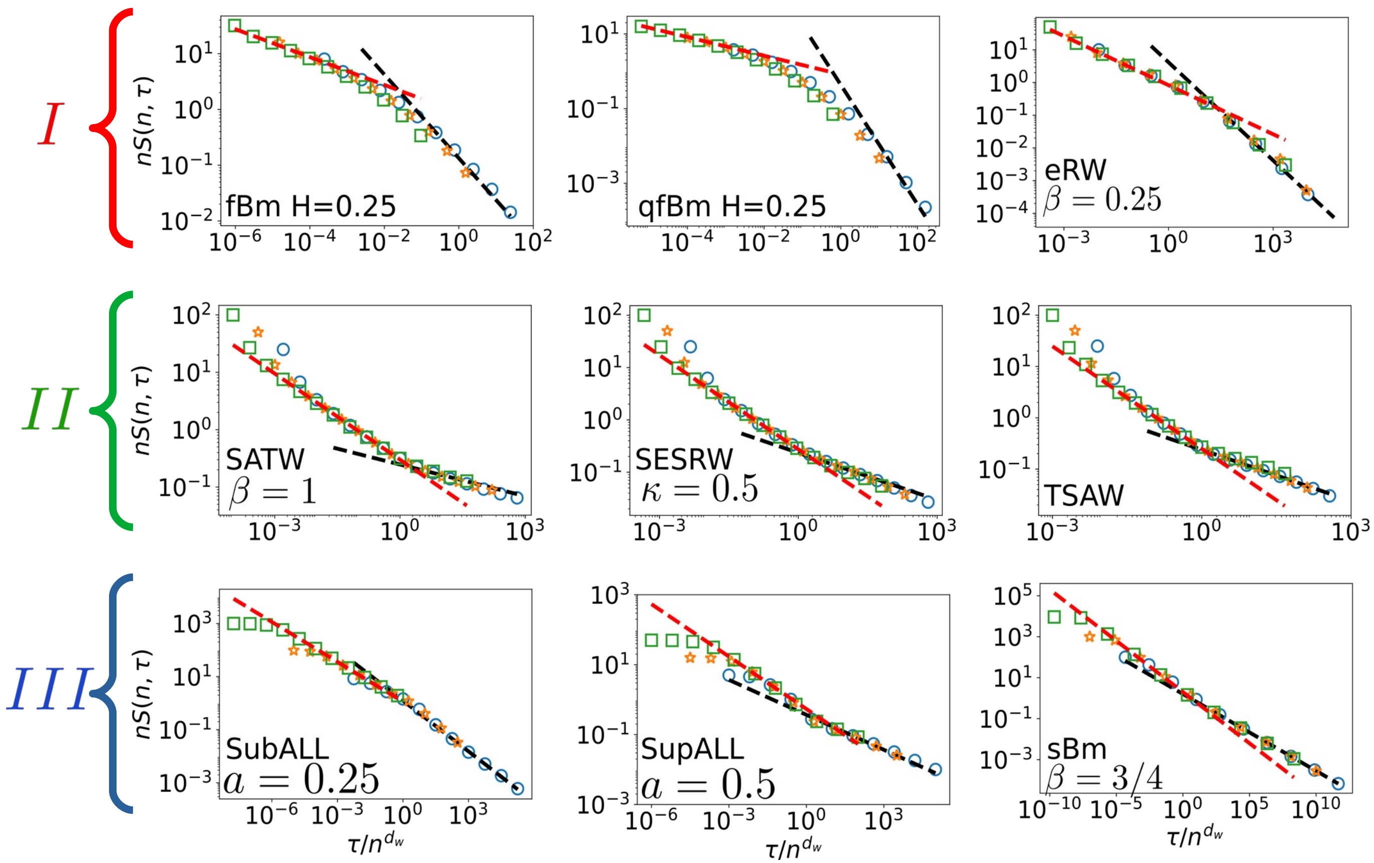

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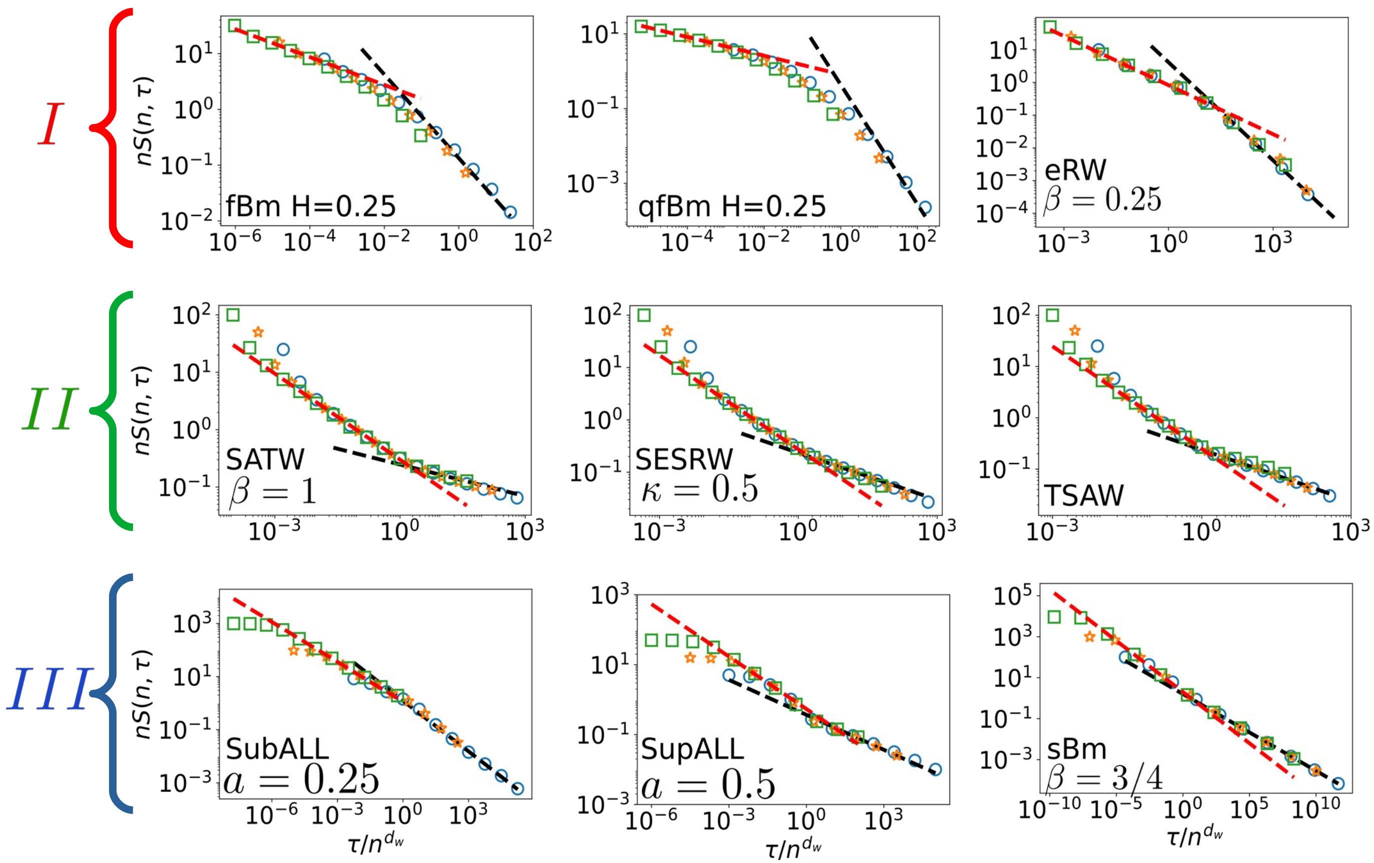


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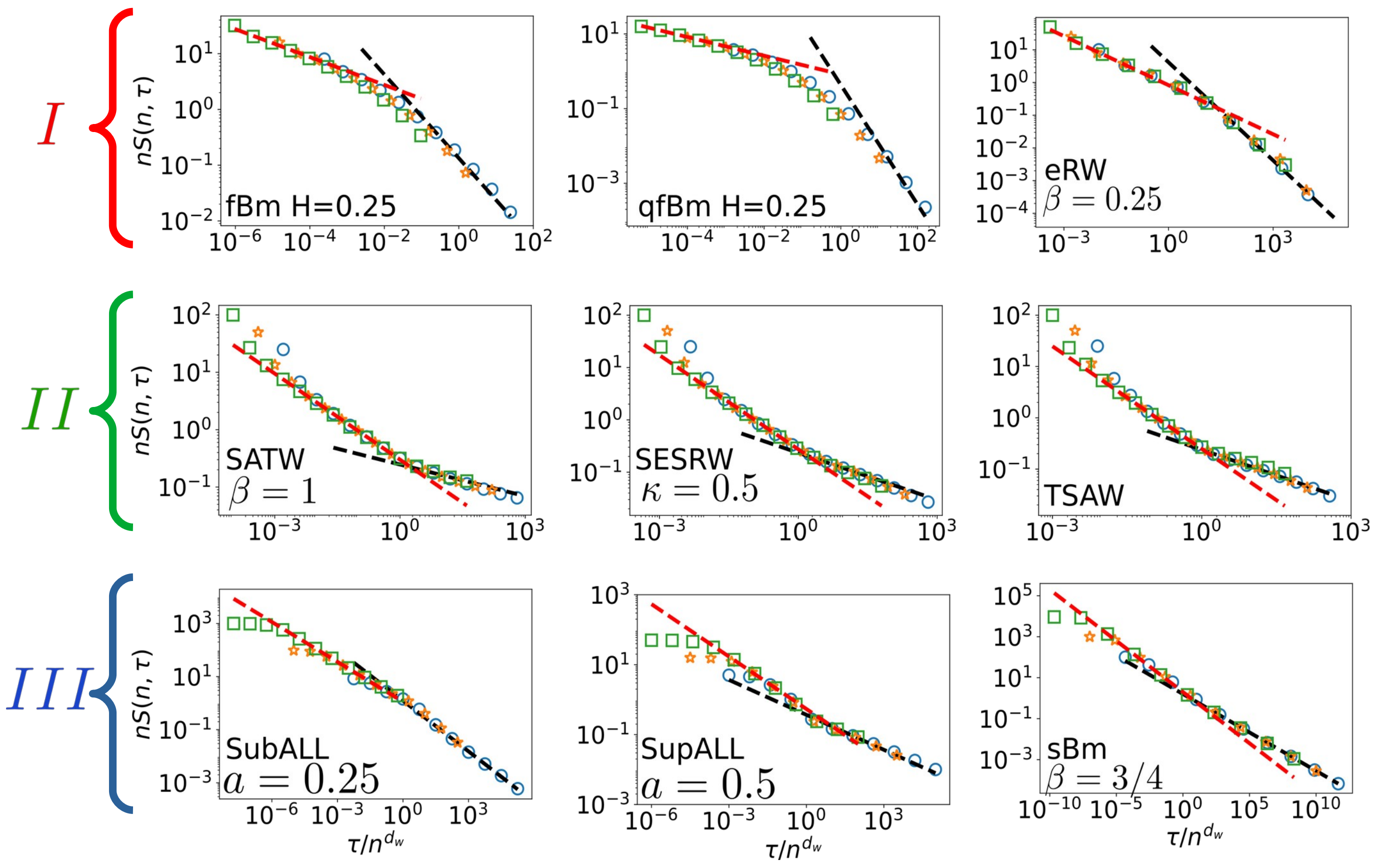


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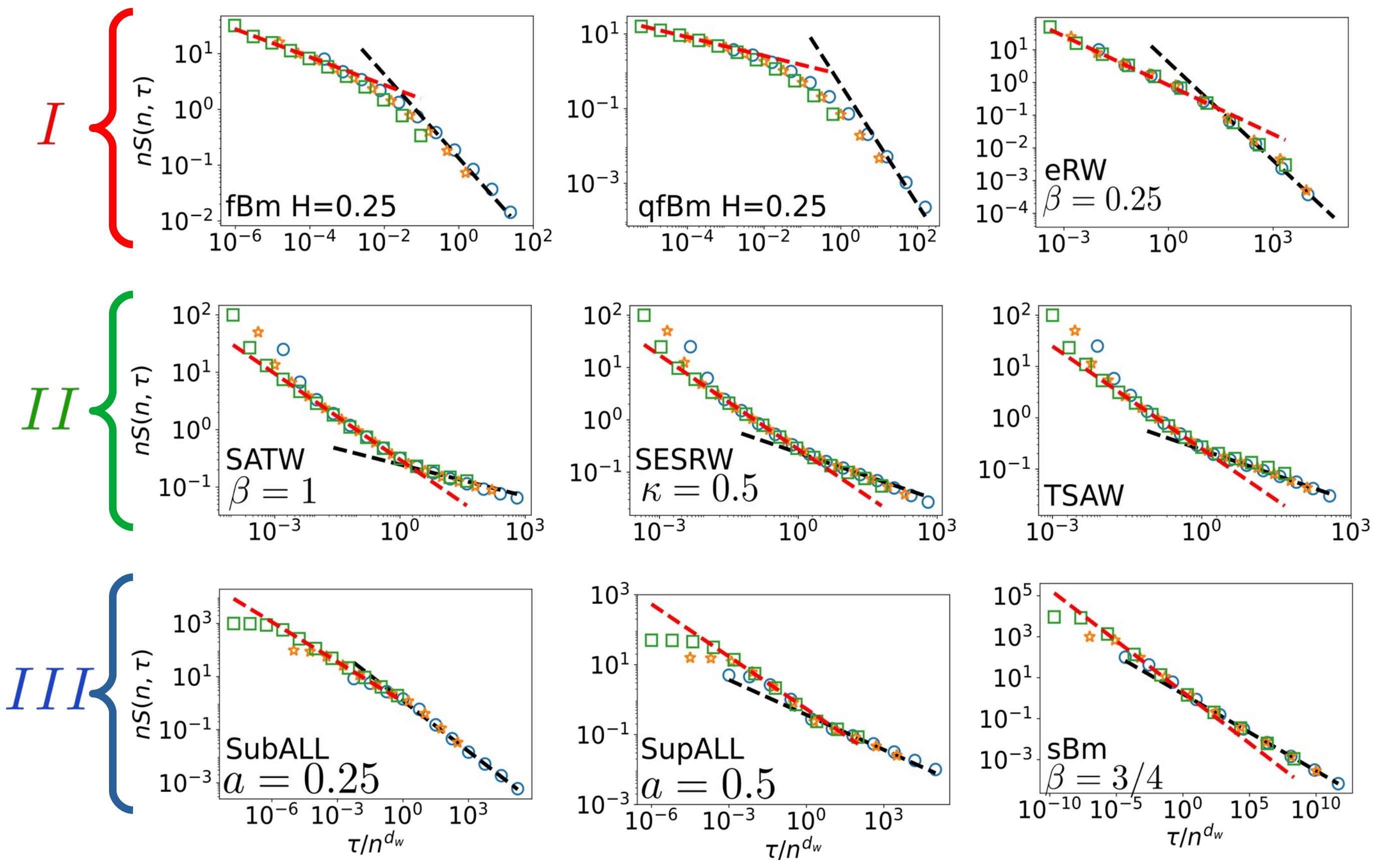


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- Very different regimes for stationary processes, $\theta = 1 - 1/d_{\mathrm{w}}^{0}$
- Early records not representatives of later ones.

# Results: comparison with biological data

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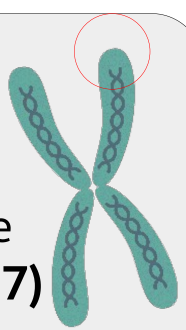
**APPLICATION:**

Displacement of a telomere
**(Stadler, *New J Phys 19,* 2017)**



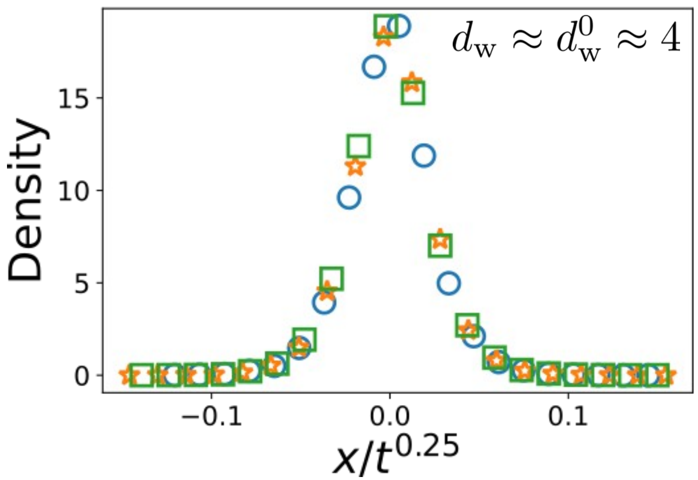
# Results: comparison with biological data

The record ages distribution **changes with the number n of records**, and has two **algebraic decays** of exponents $1/d_{\mathrm{w}}^0$ and $\theta$

$$S(n,\tau) = \mathbb{P}(\tau_n \geq \tau)$$

$$\propto \frac{1}{n}\left(\frac{n^{d_{\mathrm{w}}}}{\tau}\right)^{1/d_{\mathrm{w}}^0} \qquad \tau \ll n^{d_{\mathrm{w}}}$$

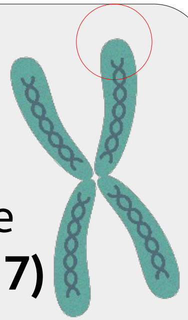
$$\propto \frac{1}{n}\left(\frac{n^{d_{\mathrm{w}}}}{\tau}\right)^{\theta} \qquad \tau \gg n^{d_{\mathrm{w}}}$$




Distribution of telomere position (t=20, 40, 80).

**APPLICATION:**

Displacement of a telomere **(Stadler, *New J Phys 19,* 2017)**



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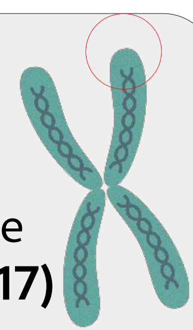

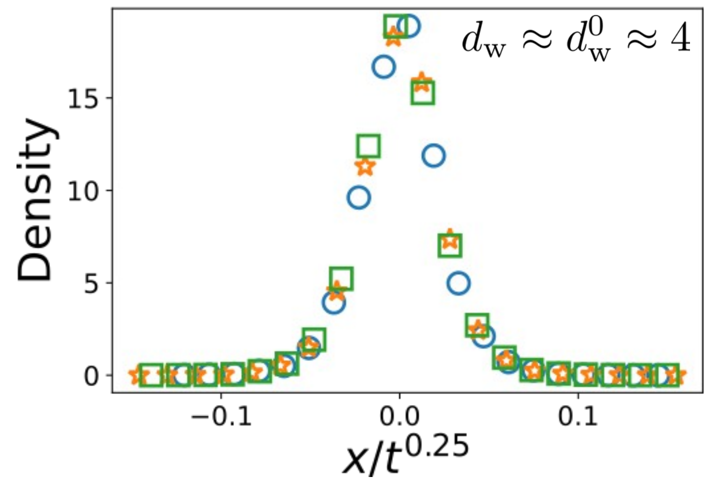


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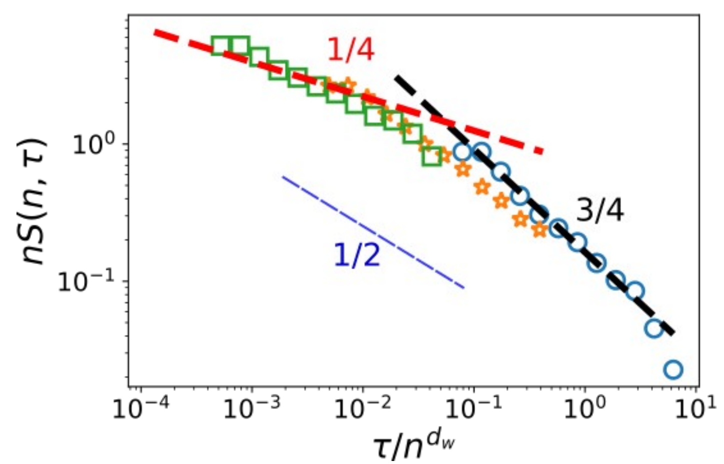


Time distribution between the $n^{th}$ and $(n+1)^{st}$ record (n=1, 3, 6).

# Conclusion

Complete caracterisation of records of 1d scale invariant non-Markovian random walks,

- With memory (FBM)
- Interacting with its environment (SATW)
- Time and space dependent dynamic

**Records for a very large class of systems!**

Relevant to describe the complex motion of biological tracers.

# MDCK cell